Derivative of Inverse Hyperbolic functions : Examples

$$y = \tanh^{-1}(\sinh x) + \sinh^{-1}(\operatorname{sech} x)$$

$$\frac{dy}{dx} = \frac{1}{1-\sinh^2 x} \cdot \frac{d}{dx}(\sinh x) + \frac{1}{\sqrt{1+\operatorname{sech}^2 x}} \frac{d}{dx}(\operatorname{sech} x)$$

$$= \frac{1}{1-\sinh^2 x} \cdot \cosh x + \frac{1}{\sqrt{1+\frac{1}{\cosh^2 x}}} \cdot (-\operatorname{sech} x \tanh x)$$

$$= \frac{\cosh x}{1-\sinh^2 x} - \frac{\operatorname{sech} x \tanh x}{\sqrt{\cosh^2 x + 1}} \cdot \cosh x$$

$$= \frac{\cosh x}{1-\sinh^2 x} - \frac{\tanh x}{\sqrt{\cosh^2 x+1}} \quad \text{Ans.}$$

② $y = \tanh^{-1}\left(\frac{2x}{1+x^2}\right)$

$$\frac{dy}{dx} = \frac{1}{1-\left(\frac{2x}{1+x^2}\right)^2} \frac{d}{dx}\left(\frac{2x}{1+x^2}\right)$$

$$= \frac{1}{(1+x^2)^2-(2x)^2} \cdot \cancel{(1+x^2)^2} \cdot \frac{(1+x^2)\cdot 2 - (2x)(2x)}{\cancel{(1+x^2)^2}}$$

$$= \frac{2+2x^2-4x^2}{1+x^4+2x^2-4x^2}$$

$$= \frac{2(1-x^2)}{(1-x^2)^2} = \frac{2}{1-x^2} \quad \underline{\text{Ans}}$$

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3. $$y = \log\sqrt{x^2-1} - x\tanh^{-1}x$$

$$= \log(x^2-1)^{1/2} - x\tanh^{-1}x$$

$$y = \frac{1}{2}\log(x^2-1) - x\tanh^{-1}x$$

$$\frac{dy}{dx} = \frac{1}{2}\left[\frac{1}{x^2-1}\frac{d}{dx}(x^2-1)\right] - \left\{x\cdot\frac{1}{1-x^2} + \tanh^{-1}x\cdot(1)\right]$$

$$= \frac{1}{2}\left[\frac{1}{(x^2-1)}(2x)\right] - \left\{\frac{x}{1-x^2} + \tanh^{-1}x\right]$$

$$= \frac{x}{x^2-1} - \frac{x}{1-x^2} - \tanh^{-1}x$$

$$= \frac{x}{x^2-1} + \frac{x}{x^2-1} - \tanh^{-1}x$$

$$= \frac{2x}{x^2-1} - \tanh^{-1}x \quad \text{Ans}$$