

## Derivative of Inverse Hyperbolic functions : Examples

$$y = \tanh^{-1}(\sinh x) + \sinh^{-1}(\operatorname{sech} x)$$

$$\frac{dy}{dx} = \frac{1}{1 - \sinh^2 x} \cdot \frac{d}{dx}(\sinh x) + \frac{1}{\sqrt{1 + \operatorname{sech}^2 x}} \frac{d}{dx}(\operatorname{sech} x)$$

$$= \frac{1}{1 - \sinh^2 x} \cdot \cosh x + \frac{1}{\sqrt{1 + \frac{1}{\cosh^2 x}}} \cdot (-\operatorname{sech} x \tanh x)$$

$$= \frac{\cosh x}{1 - \sinh^2 x} - \frac{\operatorname{sech} x \tanh x}{\sqrt{\cosh^2 x + 1}} \cdot \cosh x$$

$$= \frac{\cosh x}{1 - \sinh^2 x} - \frac{\tanh x}{\sqrt{\cosh^2 x + 1}} \quad \text{Ans.} \\ =$$

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$$\textcircled{2} \quad y = \tanh^{-1} \left( \frac{2x}{1+x^2} \right)$$

$$\frac{dy}{dx} = \frac{1}{1 - \left( \frac{2x}{1+x^2} \right)^2} \cdot \frac{d}{dx} \left( \frac{2x}{1+x^2} \right)$$

$$= \frac{1}{(1+x^2)^2 - (2x)^2} \cdot \frac{\cancel{(1+x^2)^2} \cdot (1+x^2) \cdot 2 - (2x)(2x)}{\cancel{(1+x^2)^2}}$$

$$= \frac{2 + 2x^2 - 4x^2}{1 + x^4 + 2x^2 - 4x^2}$$

$$= \frac{2(1 - x^2)}{(1 - x^2)^2} = \frac{2}{1 - x^2} \quad \underline{\text{Ans}}$$

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③

$$y = \log \sqrt{x^2 - 1} - x \tanh^{-1} x$$

$$= \log (x^2 - 1)^{1/2} - x \tanh^{-1} x$$

$$y = \frac{1}{2} \log(x^2 - 1) - x \tanh^{-1}x$$

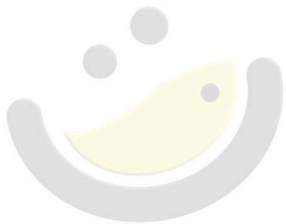
$$\frac{dy}{dx} = \frac{1}{2} \left[ \frac{1}{x^2 - 1} \frac{d}{dx} (x^2 - 1) \right] - \left[ x \frac{1}{1 - x^2} + \tanh^{-1}x \cdot (1) \right]$$

$$= \frac{1}{2} \left[ \frac{1}{(x^2 - 1)} (2x) \right] - \left[ \frac{x}{1 - x^2} + \tanh^{-1}x \right]$$

$$= \frac{x}{x^2 - 1} - \frac{x}{1 - x^2} - \tanh^{-1}x$$

$$= \frac{x}{x^2-1} + \frac{x}{x^2-1} - \tanh^{-1}x$$

$$= \frac{2x}{x^2-1} - \tanh^{-1}x \quad \underline{\text{Ans}}$$



OMG { MATHS }

The poetry of logical ideas.