

## Derivative of Inverse Hyperbolic functions : Example

$$y = \frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \sinh^{-1}\left(\frac{x}{a}\right)$$

$$\frac{dy}{dx} = \frac{1}{2} \left[ x \cdot \frac{1}{\sqrt{x^2 + a^2}} \frac{d}{dx} (x^2 + a^2) + \sqrt{x^2 + a^2} \cdot (1) \right]$$

$$+ \frac{a^2}{2} \cdot \frac{1}{\sqrt{1 + \frac{x^2}{a^2}}} \frac{d}{dx} \frac{x}{a}$$

$$= \frac{1}{2} \left[ \frac{x}{\sqrt{x^2 + a^2}} \cdot (2x) + \sqrt{x^2 + a^2} \right] + \frac{a^2}{2} \left[ \frac{1}{\sqrt{\frac{a^2 + x^2}{a^2}}} \right] \cdot \frac{1}{a}$$

$$= \frac{1}{2} \left[ \frac{x^2}{\sqrt{x^2+a^2}} + \sqrt{x^2+a^2} \right] + \frac{a^2}{2} \left[ \frac{\cancel{a}}{\sqrt{a^2+x^2}} \right] \cdot \frac{1}{\cancel{a}}$$

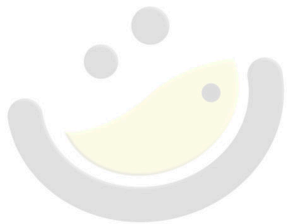
$$= \frac{1}{2} \left[ \frac{x^2 + x^2 + a^2}{\sqrt{x^2+a^2}} \right] + \frac{a^2}{2} \cdot \frac{1}{\sqrt{x^2+a^2}}$$

$$= \frac{1}{2\sqrt{x^2+a^2}} \left[ x^2 + x^2 + a^2 + a^2 \right]$$

$$= \frac{1}{2\sqrt{x^2+a^2}} \left[ 2x^2 + 2a^2 \right] = \frac{1}{\cancel{2}\sqrt{x^2+a^2}} \cdot \cancel{2} (x^2+a^2)$$

$$= \sqrt{x^2 + 4^2}$$

Ans



**OMG { MATHS }**  
The poetry of logical ideas.