Derivative of Inverse Hyperbolic functions : Example

$$y = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\sinh^{-1}\left(\frac{x}{a}\right)$$

$$\frac{dy}{dx} = \frac{1}{2}\left[x \cdot \frac{1}{2\sqrt{x^2+a^2}}\frac{d}{dx}(x^2+a^2) + \sqrt{x^2+a^2}.(1)\right]$$

$$+ \frac{a^2}{2} \cdot \frac{1}{\sqrt{1+\frac{x^2}{a^2}}}\frac{d}{dx}\frac{x}{a}.$$

$$= \frac{1}{2}\left[\frac{x}{2\sqrt{x^2+a^2}} \cdot (2x) + \sqrt{x^2+a^2}\right] + \frac{a^2}{2}\left[\frac{1}{\sqrt{\frac{a^2+x^2}{a^2}}}\right] \cdot \frac{1}{a}$$

$$= \frac{1}{2}\left[\frac{x^2}{\sqrt{x^2+a^2}} + \sqrt{x^2+a^2}\right] + \frac{a^2}{2}\left[\frac{\cancel{a}}{\sqrt{a^2+x^2}}\right]\cdot\frac{1}{\cancel{a}}$$

$$= \frac{1}{2}\left[\frac{x^2 + x^2+a^2}{\sqrt{x^2+a^2}}\right] + \frac{a^2}{2}\cdot\frac{1}{\sqrt{x^2+a^2}}$$

$$= \frac{1}{2\sqrt{x^2+a^2}}\left[x^2 + x^2 + a^2 + a^2\right]$$

$$= \frac{1}{2\sqrt{x^2+a^2}}\left[2x^2 + 2a^2\right] = \frac{1}{\cancel{2}\sqrt{x^2+a^2}}\cdot\cancel{2}\left(x^2+a^2\right)$$

$$= \sqrt{x^2 + a^2}$$ Ans