Derivative of Hyperbolic functions : Example

$$y = x^{\cosh x} + (\sinh x)^{2x}$$

$$y = u + v$$

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \quad \text{— (1)}$$

$$u = x^{\cosh x}$$

Taking Log Both sides

$$\log u = \log (x)^{\cosh x}$$

$$\log u = \cosh x \cdot \log x$$

$$\frac{1}{u}\frac{du}{dx} = \cosh x\left(\frac{1}{x}\right) + \log x\,(\sinh x)$$

$$\frac{du}{dx} = u\left[\frac{\cosh x}{x} + \log x \cdot \sinh x\right]$$

$$= x^{\cosh x}\left[\frac{\cosh x}{x} + \log x \cdot \sinh x\right] \quad -(11)$$

$$v = (\sinh x)^{2x}$$

Taking log Both sides

$$\log v = \log (\sinh x)^{2x}$$

$$\log v = 2x \cdot \log (\sinh x)$$

Diff. both sides

$$\frac{1}{v}\frac{dv}{dx} = 2\left[x \cdot \frac{1}{\sinh x}\frac{d}{dx}(\sinh x) + \log(\sinh x)(1)\right]$$

$$\frac{dv}{dx} = 2v\left[\frac{x}{\sinh x} \cdot \cosh x + \log(\sinh x)\right]$$

$$= 2(\sinh x)^{2x}\left[x\coth x + \log(\sinh x)\right] - (11)$$

Now from (9), (10) and (11)

$$\frac{dy}{dx} = x^{\cosh x}\left[\frac{\cosh x}{x} + \log x \sinh x\right] + 2(\sinh x)^{2x}\left[x\coth x + \log(\sinh x)\right]$$

thus.