Calculus : 55

Derivative of Inverse Hyperbolic functions : Example

$$y = \sin^{-1}(\tanh x^2) + \operatorname{cosech}^{-1}(\tan x) + \sinh^{-1}(\operatorname{sech} x)$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-\tanh^2 x^2}}\frac{d}{dx}(\tanh x^2) + \frac{-1}{|\tan x|\sqrt{1+\tan^2 x}}\frac{d}{dx}\tan x$$

$$+\frac{1}{\sqrt{1+\operatorname{sech}^2 x}}\frac{d}{dx}(\operatorname{sech} x)$$

$$= \frac{1}{\sqrt{\operatorname{sech}^2 x^2}} \cdot \operatorname{sech}^2 x^2 \frac{d}{dx}(x^2) - \frac{1}{|\tan x| \sec x} \cdot \sec^2 x$$

$$+ \frac{1}{\sqrt{1+\operatorname{sech}^2 x}} \cdot (-\operatorname{sech} x \tanh x)$$

$$= \frac{1}{\operatorname{sech} x^2} \cdot \operatorname{sech}^2 x^2 \cdot 2x - \frac{\sec x}{|\tan x|} + \frac{1}{\sqrt{1+\operatorname{sech}^2 x}} \cdot (-\operatorname{sech} x \tanh x)$$

$$= 2x \operatorname{sech} x^2 - \frac{\sec x}{|\tan x|} - \frac{\operatorname{sech} x \tanh x}{\sqrt{1+\operatorname{sech}^2 x}} \quad \underline{\underline{\text{Ans.}}}$$