Derivative of Hyperbolic functions: lecture 1

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{\frac{e^x - e^{-x}}{2}}{\frac{e^x + e^{-x}}{2}} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\coth x = \frac{\cosh x}{\sinh x} = \frac{\frac{e^x + e^{-x}}{2}}{\frac{e^x - e^{-x}}{2}}$$

$$= \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

$$\operatorname{cosec} h x = \frac{1}{\sinh x} = \frac{2}{e^x - e^{-x}}$$

$$\operatorname{sech} x = \frac{1}{\cosh x} = \frac{2}{e^x + e^{-x}}$$

$$\frac{d}{dx}\sinh x = \frac{d}{dx}\left(\frac{e^x - e^{-x}}{2}\right)$$

$$= \frac{1}{2}\left(e^x - e^{-x}(-1)\right)$$

$$= \frac{1}{2}\left(e^x + e^{-x}\right) = \cosh x$$

$$\frac{d}{dx}\cosh x = \frac{d}{dx}\left(\frac{e^x + e^{-x}}{2}\right) = \frac{e^x + e^{-x}(-1)}{2}$$

$$= \frac{e^x - e^{-x}}{2} = \sinh x$$

$$\frac{d}{dx} \tanh x = \frac{d}{dx}\left(\frac{\sinh x}{\cosh x}\right)$$

$$= \frac{\cosh x \frac{d}{dx}(\sinh x) - \sinh x \frac{d}{dx}\cosh x}{\cosh^2 x}$$

$$= \frac{\cosh x \cdot \cosh x - \sinh x \cdot \sinh x}{\cosh^2 x}$$

$\sinh x = \cosh x$

$\cosh x = \sinh x$

$\tanh x = \operatorname{sech}^2 x$

$\coth x = -\operatorname{cosech}^2 x$

$\operatorname{cosech} x = -\operatorname{cosech} x \coth x$

$\operatorname{sech} x = -\operatorname{sech} x \tanh x$

$$= \frac{\cosh^2 x - \sinh^2 x}{\cosh^2 x} = \frac{1}{\cosh^2 x} = \operatorname{sech}^2 x$$

$$\frac{d}{dx} \coth x = \frac{d}{dx}\left(\frac{\cosh x}{\sinh x}\right)$$

$$= \frac{\sinh x \frac{d}{dx}(\cosh x) - \cosh x \frac{d}{dx} \sinh x}{\sinh^2 x}$$

$$= \frac{\sinh x \cdot \sinh x - \cosh x \cdot \cosh x}{\sinh^2 x}$$

$$= \frac{\sinh^2 x - \cosh^2 x}{\sinh^2 x}$$

$$= \frac{-(\cosh^2 x - \sinh^2 x)}{\sinh^2 x}$$

$$= \frac{-1}{\sinh^2 x} = -\operatorname{cosech}^2 x$$

$$\frac{d}{dx} \operatorname{Cosec} h x = d \frac{1}{dx \sinh x} = \frac{d}{dx} (\sinh x)^{-1}$$

$$= (-1)(\sinh x)^{-1-1} \frac{d}{dx} \sinh x$$

$$= -1(\sinh x)^{-2} \cdot \cosh x = \frac{-1}{\sinh^2 x} \cdot \cosh x$$

$$= \frac{-1}{\sinh x} \cdot \frac{\cosh x}{\sinh x} = -\operatorname{Cosech} x \coth x$$

$$\frac{d}{dx} \operatorname{sech} x = \frac{d}{dx} \frac{1}{\cosh x} = \frac{d}{dx} (\cosh x)^{-1}$$

$$= -(\cosh x)^{-1-1} \frac{d}{dx}(\cosh x)$$

$$= -(\cosh x)^{-2} \cdot \sinh x$$

$$= \frac{-\sinh x}{\cosh x \cdot \cosh x} = -\operatorname{sech} x \tanh x.$$