

Derivative of Hyperbolic functions : Examples

Exp 1 $y = x \sinh x$

$$\frac{dy}{dx} = \frac{d}{dx} (x \sinh x)$$

$$= x \frac{d}{dx} \sinh x + \sinh x \frac{d}{dx} x$$

$$= x \cdot \cosh x + \sinh x \cdot 1$$

$$= x \cosh x + \sinh x$$



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Exp 2

Prove that $\frac{d}{dx} [\tanh(\log x)] = \frac{4x}{(x^2+1)^2}$

L.H.S

$$\frac{d}{dx} \left[\frac{e^{\log x} - e^{-\log x}}{e^{\log x} + e^{-\log x}} \right]$$

$$\frac{d}{dx} \left[\frac{e^{\log x} - e^{\log x^{-1}}}{e^{\log x} + e^{\log x^{-1}}} \right]$$

$$\frac{d}{dx} \left[\frac{x - \frac{1}{x}}{x + \frac{1}{x}} \right] = \frac{d}{dx} \left[\frac{x^2 - 1}{x} \right] \left/ \frac{x^2 + 1}{x} \right.$$

$$\frac{d}{dx} \left(\frac{x^2 - 1}{x^2 + 1} \right)$$

$$= \frac{(x^2 + 1) \frac{d}{dx} (x^2 - 1) - (x^2 - 1) \frac{d}{dx} (x^2 + 1)}{(x^2 + 1)^2}$$

$$= \frac{(x^2 + 1)(2x) - (x^2 - 1)(2x)}{(x^2 + 1)^2}$$

$$= \frac{2x^3 + 2x - 2x^3 + 2x}{(x^2 + 1)^2} = \frac{4x}{(x^2 + 1)^2} = \underline{\underline{R.H.S}}$$

Ex 3 $y = e^{ax} \cosh(bx)$

$$\frac{dy}{dx} = e^{ax} \frac{d}{dx} \cosh(bx) + \cosh(bx) \frac{d}{dx} e^{ax}$$

$$= e^{ax} \sinh bx \cdot (b) + \cosh bx \cdot e^{ax} \cdot a$$

$$= b e^{ax} \sinh bx + a e^{ax} \cosh bx$$

$$= e^{ax} [b \sinh bx + a \cosh bx]$$

Ex 4

$$y = x \cosh x - \sinh x$$

$$\frac{dy}{dx} = x \frac{d}{dx} \cosh x + \cosh x \frac{d}{dx} (x) - \frac{d}{dx} \sinh x$$

$$= x \sinh x + \cancel{\cosh x} (1) - \cancel{\cosh x}$$

$$= x \sinh x \quad \underline{\underline{Ans}}$$



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Exps $y = x^{\tanh x}$

Taking log both sides

$$\log y = \log(x^{\tanh x})$$

$$\log y = \tanh x \log x$$

Differentiate Both sides

$$\frac{1}{y} \frac{dy}{dx} = \tanh x \frac{d}{dx} \log x + \log x \frac{d}{dx} \tanh x$$

$$\begin{aligned} \frac{dy}{dx} &= y \left[\tanh x \cdot \frac{1}{x} + \log x \cdot \operatorname{sech}^2 x \right] \\ &= x^{\tanh x} \left[\frac{\tanh x}{x} + \operatorname{sech}^2 x \cdot \log x \right] \end{aligned}$$

Ans
=

Exp 6

$$y = \log(\cosh x) + \frac{1}{2 \cosh^2 x}$$

$$\frac{dy}{dx} = \frac{1}{\cosh x} \frac{d}{dx} \cosh x + \frac{1}{2} \frac{d}{dx} (\cosh x)^{-2}$$

$$= \frac{1}{\cosh x} \cdot \sinh x + \frac{1}{2} (-2) (\cosh x)^{-3} \frac{d}{dx} \cosh x$$

$$= \tanh x + (-1) (\cosh x)^{-3} \cdot \sinh x$$

$$= \tanh x - \frac{\sinh x}{(\cosh x)^3}$$

$$= \tanh x - \tanh x \cdot \frac{1}{(\cosh x)^2}$$

$$= \tanh x [1 - \operatorname{sech}^2 x]$$

$$= \tanh x [\tanh^2 x] \quad \left[\because 1 - \operatorname{sech}^2 x = \tanh^2 x \right]$$

$$= (\tanh x)^3 = \tanh^3 x \quad \text{Ans}$$

Exp 7

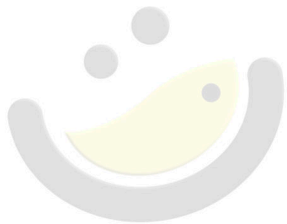
$$y = \sqrt{1 + \sinh^2 x}$$

$$= \sqrt{\cosh^2 x}$$

$$y = |\cosh x|$$

$$\frac{dy}{dx} = \frac{\cosh x}{|\cosh x|} \frac{d}{dx} (\cosh x)$$

$$= \frac{\cosh x}{|\cosh x|} \cdot \sinh x \cdot \underline{\underline{dx}}$$



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