Derivative of Hyperbolic functions : Examples

Exp 1 $y = x \sinh x$

$$\frac{dy}{dx} = \frac{d}{dx}(x \sinh x)$$

$$= x \frac{d}{dx} \sinh x + \sinh x \frac{d}{dx} x.$$

$$= x \cdot \cosh x + \sinh x \cdot 1$$

$$= x \cosh x + \sinh x$$

Exp2 Prove that $\frac{d}{dx}\left[\tanh(\log x)\right] = \frac{4x}{(x^2+1)^2}$

L.H.S

$$\frac{d}{dx}\left[\frac{e^{\log x} - e^{-\log x}}{e^{\log x} + e^{-\log x}}\right]$$

$$\frac{d}{dx}\left[\frac{e^{\log x} - e^{\log x^{-1}}}{e^{\log x} + e^{\log x^{-1}}}\right]$$

$$\frac{d}{dx}\left[\frac{x - \frac{1}{x}}{x + \frac{1}{x}}\right] = \frac{d}{dx}\left[\frac{x^2-1}{x} \Big/ \frac{x^2+1}{x}\right]$$

$$\frac{d}{dx}\left(\frac{x^2-1}{x^2+1}\right)$$

$$= \frac{(x^2+1)\frac{d}{dx}(x^2-1) - (x^2-1)\frac{d}{dx}(x^2+1)}{(x^2+1)^2}$$

$$= \frac{(x^2+1)(2x) - (x^2-1)(2x)}{(x^2+1)^2}$$

$$= \frac{2x^3+2x-2x^3+2x}{(x^2+1)^2} = \frac{4x}{(x^2+1)^2} = \text{R.H.S}$$

Ex 3 $y = e^{ax} \cosh(bx)$

$$\frac{dy}{dx} = e^{ax} \frac{d}{dx} \cosh(bx) + \cosh(bx) \frac{d}{dx} e^{ax}$$

$$= e^{ax} \sinh bx \cdot (b) + \cosh bx \cdot e^{ax} \cdot a$$

$$= b e^{ax} \sinh bx + a e^{ax} \cosh bx$$

$$= e^{ax} \{ b \sinh bx + a \cosh bx \}$$

Exp.4 $y = x\cosh x - \sinh x$

$$\frac{dy}{dx} = x\frac{d}{dx}\cosh x + \cosh x \frac{d}{dx}(x) - \frac{d}{dx}\sinh x$$

$$= x\sinh x + \cosh x\,(1) - \cosh x$$

$$= x\sinh x \quad \text{Ans}$$

EX 5 $y = x^{\tanh x}$

Taking log both sides

$$\log y = \log\left(x^{\tanh x}\right)$$

$$\log y = \tanh x \log x$$

Differentiate Both sides

$$\frac{1}{y}\frac{dy}{dx} = \tanh x \frac{d}{dx}\log x + \log x \frac{d}{dx}\tanh x$$

$$\frac{dy}{dx} = y\left[\tanh x \frac{1}{x} + \log x \cdot \operatorname{sech}^2 x\right]$$

$$= x^{\tanh x}\left[\frac{\tanh x}{x} + \operatorname{sech}^2 x \cdot \log x\right]$$

Ans

Exp 6 $y = \log(\cosh x) + \frac{1}{2\cosh^2 x}$

$$\frac{dy}{dx} = \frac{1}{\cosh x}\frac{d}{dx}\cosh x + \frac{1}{2}\frac{d}{dx}(\cosh x)^{-2}$$

$$= \frac{1}{\cosh x} \cdot \sinh x + \frac{1}{2}(-2)(\cosh x)^{-3} \frac{d}{dx} \cosh x$$

$$= \tanh x + (-1)(\cosh x)^{-3} \cdot \sinh x$$

$$= \tanh x - \frac{\sinh x}{(\cosh x)^3}$$

$$= \tanh x - \tanh x \cdot \frac{1}{(\cosh x)^2}$$

$$= \tanh x \left\{ 1 - \operatorname{sech}^2 x \right\}$$

$$= \tanh x \left[\tanh^2 x\right] \qquad \left[\because 1 - \operatorname{sech}^2 x = \tanh^2 x\right]$$

$$= (\tanh x)^3 = \tanh^3 x \quad \text{Ans}$$

Ex 7 $y = \sqrt{1 + \sinh^2 x}$

$$= \sqrt{\cosh^2 x}$$

$$y = |\cosh x|$$

$$\frac{dy}{dx} = \frac{\cosh x}{|\cosh x|} \frac{d}{dx}(\cosh x)$$

$$= \frac{\cosh x}{|\cosh x|} \cdot \sinh x \cdot dx$$