

Derivative of Hyperbolic functions : Examples

Prove that

$$\frac{d}{dx} \log (\tanh x/2) = \operatorname{cosech} x$$

L.H.S

$$\frac{d}{dx} \log (\tanh x/2)$$

$$= \frac{1}{\tanh x/2} \frac{d}{dx} \tanh x/2$$

$$= \frac{1}{\tanh x/2} \cdot \operatorname{sech}^2 x/2 \cdot \frac{d}{dx} x/2$$

$$= \frac{1}{\tanh x/2} \cdot \operatorname{sech}^2 x/2 \cdot \frac{1}{2}$$

$$= \frac{\cosh x/2}{\sinh x/2} \cdot \frac{1}{\cosh^2 x/2} \cdot \frac{1}{2}$$

$$= \frac{1}{2 \sinh x/2 \cosh x/2} = \frac{1}{\sinh x}$$

$$= \operatorname{cosech} x = \text{R.H.S} \text{ Hence Proved}$$

$$\left[\begin{array}{l} \sinh 2x = 2 \sinh x \\ \cosh x \end{array} \right]$$