Limit and Continuity

Examine the Continuity of

$$f(x) = \begin{cases} 1+x & x \leq 2 \\ 5-x & x > 2 \end{cases} \quad \text{at } x=2$$

Sol.

$$\lim_{x \to 2^-} 1+x$$

$$\lim_{h \to 0} 1+(2-h) = 3. \quad \text{--- (1)}$$

$$\lim_{x \to 2^-}$$

$$h \to 0 \quad (x = 2-h.)$$

$$\lim_{x \to 2^+} 5 - x$$

$x \to 2^+$
$x = 2+h$
$h \to 0$

$$\lim_{h \to 0} 5 - (2+h) = 5 - (2+0)$$

$$= 3 \quad \text{— (II)}$$

$$f(2) = 1 + x = 1 + 2 = 3 \quad \text{— (III)}$$

from (I), (II) and (III)

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} f(x) = f(2).$$

Hence By def.

$f(x)$ is continous

$$f(x) = \begin{cases} \dfrac{|x|+x}{3} & x \leq 3 \\[2ex] \dfrac{2|x-3|}{x-3} & x > 3 \end{cases}$$

Sol.

$$\lim_{x \to 3^-} \frac{|x|+x}{3}$$

$$= \lim_{h \to 0} \frac{|3-h| + (3-h)}{3}$$

$x \to 3^-$
$x = 3-h.$
$h \to 0$

$$= \frac{|3| + 3}{3} = \frac{3+3}{3} = \frac{6}{3} = 2 \quad \text{— (I)}$$

$$\lim_{x \to 3^+} \frac{2|x-3|}{x-3}$$

$x \to 3^+$

$x = 3 + h$

$h \to 0$

$$\lim_{h \to 0} \frac{2|\cancel{3}+h-\cancel{3}|}{(3+h-3)} = \frac{2\cancel{h}}{\cancel{h}} = 2 \quad \text{— (II)}$$

$$f(3) = \frac{|x|+x}{3} = \frac{|3|+3}{3} = \frac{6}{3} = 2 \quad \text{— (III)}$$

from (i), (ii) and (iii)

$$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x) = f(3) = 2.$$

Hence By def.

$f(x)$ is Continous at $x = 3$.