

Limit and Continuity

Examine the continuity of

$$f(x) = \begin{cases} x \sin 1/x - 1 & x \neq 0 \\ 0 & x = 0 \end{cases}$$

at $x=0$

If discontinuous, give the nature discontinuity.

Sol

$$g(x) = x \quad h(x) = \sin 1/x.$$

$$\lim_{x \rightarrow 0} g(x) = \lim_{x \rightarrow 0} x = 0$$

$$\lim_{x \rightarrow 0} |h(x)| = \lim_{x \rightarrow 0} |\sin|x|| \leq 1 \quad x \neq 0$$

$\Rightarrow h(x)$ is bounded

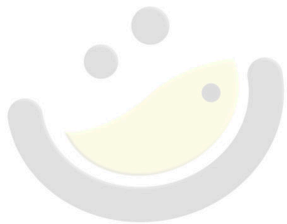
$$\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} g(x) h(x) =$$

$$= 0 \cdot (-1) = -1.$$

$$f(0) = 0.$$

$\Rightarrow \lim_{x \rightarrow 0} f(x) \neq f(0) \Rightarrow f(x)$ is discontinuous

Removable discontinuity.



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The poetry of logical ideas.