Limit and Continuity

Examine the Continuity of the function at $x = 0$

$$f(x) = \begin{cases} \sin^{-1}|x| & x \neq 0 \\ 0 & x = 0 \end{cases}$$

Sol.

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \sin^{-1}|x|$$

$$= \lim_{x \to 0^-} \sin^{-1}(-x) = \lim_{x \to 0^-} \left(-\sin^{-1}x\right)$$

$$\lim_{t \to 0^-} (-t) = \lim_{h \to 0} -(0-h)$$

$$= 0 \qquad ①$$

$$\lim_{x \to 0^+} \sin^{-1}|x) = \lim_{x \to 0^+} \sin^{-1}x$$

$$= \lim_{t \to 0^+} t$$

$$= \lim_{h \to 0} (0+h) = 0$$

$$\sin^{-1}x = t$$
$$x = \sin t$$

$$\lim_{x \to 0^-} \quad t \to 0^-$$
$$t \to 0^- \quad h \to 0$$
$$t = 0-h$$

$$\sin^{-1}x = t$$

$$x = \sin t$$
$$x \to 0^+ \quad t \to 0^+$$
$$x \to 0^+ \quad h \to 0$$
$$t = 0+h$$

$$\lim_{x \to 0^+} \sin^{-1}|x| = 0 \qquad \text{—②}$$

from ① & ②

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) \qquad \text{—③}$$

Now $f(0) = 0$ —④

from ③ & ④

$$L.H.L = R.H.L = f(0)$$

By def. $f(x)$ is Continuous.