

## Limit and Continuity

EXamine the continuity of the function

at  $x = 0$

$$f(x) = \begin{cases} \frac{\sqrt{1 - \cos 2x}}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

Sol.

$$\begin{aligned} f(x) &= \frac{\sqrt{1 - \cos 2x}}{x} & x \neq 0 \\ &= \frac{\sqrt{2 \sin^2 x}}{x} & x \neq 0 \end{aligned}$$

$$f(x) = \frac{\sqrt{2} |\sin x|}{x} \quad x \neq 0$$

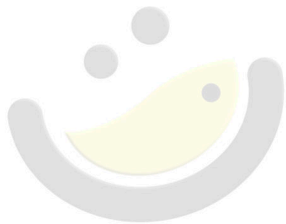
$\lim_{x \rightarrow 0^-}$

$$f(x) = \lim_{x \rightarrow 0^-} \frac{\sqrt{2} |\sin x|}{x}$$

$$= \lim_{h \rightarrow 0} \frac{\sqrt{2} |\sin(0-h)|}{(0-h)}$$

$$\left| \begin{array}{l} x \rightarrow 0^- \\ h \rightarrow 0 \end{array} \right.$$

$$= \frac{\sqrt{2} \lim_{h \rightarrow 0} |\sin(-h)|}{-h}$$



$$= -\sqrt{2} \lim_{h \rightarrow 0} \frac{\sinh}{h}$$

$$\left| \lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1 \right.$$

$$= -\sqrt{2} (1) = \underline{-\sqrt{2}} \quad \text{--- (i)}$$

$$\lim_{x \rightarrow 0^+}$$

$$f(x) = \lim_{x \rightarrow 0^+} \frac{\sqrt{2} |\sin x|}{x}$$

$$\left| \begin{array}{l} x \rightarrow 0^+ \\ h \rightarrow 0 \\ x = 0+h \end{array} \right.$$

$$= \lim_{h \rightarrow 0} \frac{\sqrt{2} |\sin(0+h)|}{(0+h)}$$

$$= \sqrt{2} \lim_{h \rightarrow 0} \frac{\sinh}{h} = \sqrt{2} \cdot 1 = \sqrt{2} \quad \text{--- (ii)}$$



from ① & ②

$$\lim_{x \rightarrow 0^-} f(x) \neq \lim_{x \rightarrow 0^+} f(x)$$

Discontinuous

Discontinuity of first kind.



OMG { MATHS }  
The poetry of logical ideas.