

Limit and Continuity

Is the function $f(x) = \begin{cases} n-1, & x=1 \\ \frac{1-x^n}{1-x}, & x \neq 1 \end{cases} \quad n \in \mathbb{N}$.

Continuous at $x=1$

If discontinuous, then state the kind of discontinuity.

Sol.

$$\lim_{x \rightarrow 1} \frac{1-x^n}{1-x} = \lim_{x \rightarrow 1} \frac{\cancel{(1-x)}(1+x+x^2+\dots+x^{n-1})}{\cancel{(1-x)}}$$

$$= 1 + 1 + 1 \dots n \text{ terms}$$

$$\lim_{x \rightarrow 1} f(x) = n$$

$$f(1) = n-1$$

$$\lim_{x \rightarrow 1} f(x) \neq f(1)$$

$\therefore f(x)$ is Discontinuous

Removable discontinuity