

Limit and Continuity : Infinite Limits

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using def Prove that

$$\lim_{x \rightarrow -\infty} \frac{4x+3}{2-5x} = \frac{-4}{5}$$

Sol.

$$f(x) = \frac{4x+3}{2-5x} \quad l = \frac{-4}{5}$$

$$|f(x) - l| = \left| \frac{4x+3}{2-5x} - \left(\frac{-4}{5} \right) \right|$$

$$= \left| \frac{20x + 15 + 8 - 20x}{5(2-5x)} \right| = \frac{23}{5(2-5x)}$$

$$|f(x) - l| < \epsilon \quad \text{when}$$

$$\frac{23}{5(2-5x)} < \epsilon$$

$$\Rightarrow \frac{23}{2-5x} < 5\epsilon$$

$$\Rightarrow \frac{1}{2-5x} < \frac{5\epsilon}{23}$$

$$\Rightarrow 2-5x > \frac{23}{5\epsilon}$$

$$\Rightarrow -5x > \frac{23}{5\epsilon} - 2$$

$$\Rightarrow x < \frac{-1}{5} \left(\frac{23}{5\epsilon} - 2 \right)$$

Thus By given $\epsilon > 0$ $\exists M = \frac{1}{5} \left(\frac{23}{5\epsilon} - 2 \right) > 0$ s.t.

$$|f(x) - 2| < \epsilon \quad \forall x < -M$$

$$\Rightarrow \lim_{x \rightarrow 0} f(x) = \frac{-4}{5}$$

$$\Rightarrow \lim_{x \rightarrow -\infty} \frac{4x+3}{2-5x} = \frac{-4}{5}$$

Hence Proved