

Limit and Continuity : Infinite Limits

Exp.

Using def Prove that

$$\lim_{x \rightarrow \infty} \frac{x}{2x+5} = \frac{1}{2}$$

Sol $f(x) = \frac{x}{2x+5}$ $l = 1/2$

$$|f(x) - l| = \left| \frac{x}{2x+5} - \frac{1}{2} \right|$$

$$\lim_{x \rightarrow \infty} f(x) = l.$$

for

$$|f(x) - l| < \epsilon \quad \forall x > M.$$

$$= \left| \frac{2x - 2x - 5}{2(2x+5)} \right| = \left| \frac{-5}{2(2x+5)} \right|$$

$$|f(x) - L| = \frac{5}{2(2x+5)} \quad \text{--- ①}$$

from ①

$$|f(x) - L| < \epsilon \quad \text{when} \quad \frac{5}{2(2x+5)} < \epsilon$$

$$\frac{5}{2x+5} < 2\epsilon$$

$$2x + 5 > \frac{5}{2\epsilon}$$

$$2x > \frac{5}{2\epsilon} - 5$$

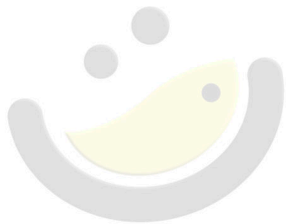
$$x > \left(\frac{5}{2\epsilon} - 5\right) \frac{1}{2} = M.$$

$|f(x) - l| < \epsilon$ for $x > M$ $M > 0$ real No.

\therefore By def. of limit $\lim_{x \rightarrow \infty} f(x) = l$

$$\lim_{x \rightarrow \infty} \frac{x}{2x+5} = \frac{1}{2}$$

Hence Proved.



OMG { MATHS }
The poetry of logical ideas.