Limit and Continuity

Prove that $\lim\limits_{x \to a} \log x = \log a$

where a is a +ve real No.

Proof $|\log x - \log a| < \epsilon$ [ϵ is however small]

$-\epsilon < \log x - \log a < \epsilon$

$-\epsilon < \log\left(\frac{x}{a}\right) < \epsilon$

$e^{-\epsilon} < \frac{x}{a} < e^{\epsilon}$

$$a e^{-\epsilon} < x < a e^{\epsilon}$$

$$a e^{-\epsilon} - a < x - a < a e^{\epsilon} - a$$

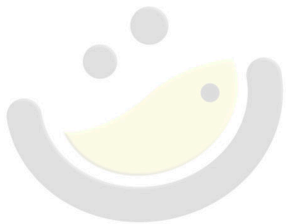$$-(a - a e^{-\epsilon}) < x - a < (a e^{\epsilon} - a)$$

let $\delta = \text{Min} \{-(a - a e^{-\epsilon}), (a e^{\epsilon} - a)\}$

$$|x - a| < \delta.$$

$$|\log x - \log a| < \epsilon \quad \text{for} \quad |x - a| < \delta$$

By def. of Limit

$$\lim_{x \to a} \log x = \log a.$$