

Gauss Test for Convergence

3. D' Alembert's Ratio Test

Let $\sum_{n=1}^{\infty} u_n$ be a positive term series such that

$$\lim_{n \rightarrow \infty} \frac{u_{n+1}}{u_n} = l$$

Then (i) $\sum_{n=1}^{\infty} u_n$ converges if $l < 1$

(ii) $\sum_{n=1}^{\infty} u_n$ diverges if $l > 1$

(iii) Test fails if $l = 1$

4. Raabe's Test

Let $\sum_{n=1}^{\infty} u_n$ be a positive term series such that

$$\lim_{n \rightarrow \infty} n \left(\frac{u_n}{u_{n+1}} - 1 \right) = l$$

Then (i) $\sum_{n=1}^{\infty} u_n$ converges if $l > 1$

(ii) $\sum_{n=1}^{\infty} u_n$ diverges if $l < 1$

(iii) Test fails if $l = 1$

Gauss Test :- $\sum u_n$ be a series of +ve terms s.t

$$\frac{u_n}{u_{n+1}} = \alpha + \frac{\beta}{n} + \frac{\gamma_n}{n^p}$$

$\alpha > 0$ $\rho > 1$ $\langle r_n \rangle$ Bounded.

If $\alpha \neq 1$

$\alpha > 1$ $\sum u_n$ Cgt

$\alpha < 1$ $\sum u_n$ divergent

If $\alpha = 1$.

$\beta > 1$ $\sum u_n$ Cgt

$\beta \leq 1$ $\sum u_n$ Divergent.

Exp

$$u_n = \frac{2^2 \cdot 4^2 \cdots (2n-2)^2}{1^2 \cdot 3^2 \cdots (2n-1)^2}$$

$$u_{n+1} = \frac{2^2 \cdot 4^2 \cdots (2n-2)^2 (2n)^2}{1^2 \cdot 3^2 \cdots (2n-1)^2 (2n+1)^2}$$

$$\lim_{n \rightarrow \infty} \frac{u_{n+1}}{u_n} = \lim_{n \rightarrow \infty} \frac{2^2 \cdot 4^2 \cdots (2n-2)^2 (2n)^2}{1^2 \cdot 3^2 \cdots (2n-1)^2 (2n+1)^2} \times \frac{1^2 \cdot 3^2 \cdots (2n-1)^2}{2^2 \cdot 4^2 \cdots (2n-2)^2}$$

$$\lim_{n \rightarrow \infty} \frac{(2n)^2}{(2n+1)^2} = \lim_{n \rightarrow \infty} \frac{\cancel{(2n)}^2}{\cancel{(2n)}^2 \left[1 + \frac{1}{2n}\right]^2} = 1.$$

\Rightarrow Ratio test fails to find the convergence

$$\lim_{n \rightarrow \infty} n \left(\frac{u_n}{u_{n+1}} - 1 \right) = \lim_{n \rightarrow \infty} n \left[\frac{2^2 \cdot 4^2 \dots (2n-2)^2}{1^2 \cdot 3^2 \dots (2n-1)^2} \times \frac{1^2 \cdot 3^2 \dots (2n-1)^2 (2n+1)^2}{2^2 \cdot 4^2 \dots (2n-2)^2 (2n)^2} - 1 \right]$$

$$= \lim_{n \rightarrow \infty} n \left[\frac{(2n+1)^2}{(2n)^2} - 1 \right]$$

$$= \lim_{n \rightarrow \infty} n \left[\frac{\cancel{4n^2} + 1 + 4n - \cancel{4n^2}}{4n^2} \right]$$

$$= \lim_{n \rightarrow \infty} \cancel{n} \cdot \cancel{n} \left[\frac{4 + 1/n}{\cancel{4n^2}} \right] = 1.$$

$$\lim_{n \rightarrow \infty} n \left[\frac{u_n}{u_{n+1}} - 1 \right] = 1.$$

Rabbe's Test fail.

Gauss Test

$$\begin{aligned} \frac{u_n}{u_{n+1}} &= \frac{2^2 \cdot 4^2 \cdots (2n-2)^2}{1^2 \cdot 3^2 \cdots (2n-1)^2} \times \frac{1^2 \cdot 3^2 \cdots (2n-1)^2 (2n)^2}{2^2 \cdot 4^2 \cdots (2n-2)^2 (2n)^2} \\ &= \frac{(2n+1)^2}{(2n)^2} = \frac{4n^2 + 1 + 4n}{4n^2} \end{aligned}$$

$$= \frac{\cancel{4n^2}}{\cancel{4n^2}} + \frac{1}{4n^2} + \frac{\cancel{4n}}{\cancel{4n^2}}$$

$$= 1 + \frac{1}{n} + \frac{1}{4n^2}$$

Compare with

$$\alpha + \frac{\beta}{n} + \frac{\gamma_n}{n^p}$$

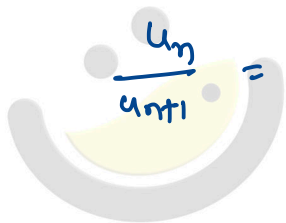
$$\alpha = 1 > 0$$

$$p > 1$$

$$p = 2 > 1$$

$\langle \gamma_n \rangle$ Bounded

$\gamma_n = \frac{1}{4}$ Bounded.

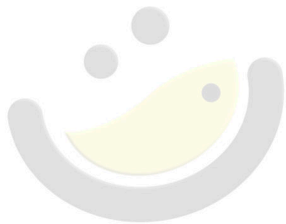


Now $\alpha = 1$.

$$\beta = 1.$$

\Rightarrow By Gauss Test

$\sum u_n$ is Divergent.



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