Find glb and lub of the set

$y = a \sin x + b \cos x + c$

Let $a = r \cos\alpha$ — (I)

$b = r \sin\alpha$ — (II)

By squaring and adding (I) & (II)

$a^2 + b^2 = r^2 (\cos^2\alpha + \sin^2\alpha)$ $[\cos^2\theta + \sin^2\theta = 1]$

$a^2 + b^2 = r^2 \Rightarrow r = \sqrt{a^2 + b^2}$ — (III)

$$y = r\cos\alpha \sin x + r\sin\alpha \cos x + c$$

$$= r\left[\cos\alpha \sin x + \sin\alpha \cos\alpha\right] + c$$

$$= r\left[\sin(x+\alpha)\right] + c \qquad \left[\sin(a+b) = \sin a\cos b + \cos a \sin b\right]$$

$$= \sqrt{a^2+b^2}\left[\sin(x+\alpha)\right] + c \quad [\text{from (11)}]$$

Now $-1 \leq \sin(x+\alpha) \leq 1$

$$-\sqrt{a^2+b^2} \leq \sqrt{a^2+b^2}\sin(x+\alpha) \leq \sqrt{a^2+b^2}$$

Trigonometric table

θ	0°	30°	45°	60°	90°	180°	270°	360°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	0	1
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined	0	Not defined	0
cosec θ	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	Not defined	-1	Not defined
sec θ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined	-1	Not defined	1
cot θ	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	Not defined	0	Not defined

$$-\sqrt{a^2+b^2}+c \leq \sqrt{a^2+b^2}\sin(x+\alpha)+c \leq \sqrt{a^2+b^2}+c$$

$$-\sqrt{a^2+b^2}+c \leq y \leq \sqrt{a^2+b^2}+c.$$

So $\quad g.l.b = c-\sqrt{a^2+b^2}$

$\quad l.u.b = c+\sqrt{a^2+b^2}$