Limit and Continuity

Example

Prove that $\lim_{x \to -\frac{5}{2}} \frac{1}{2x+5}$ does not exist.

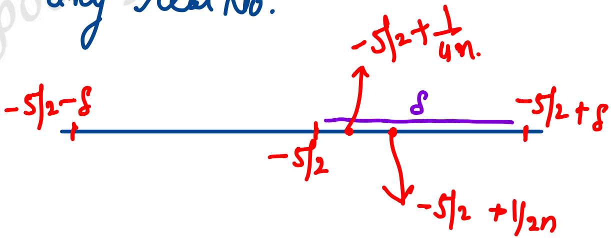
Proof

$\epsilon > 0$ be any small no. $0 < \epsilon < 1$.

$\delta > 0$ be any real No.

Let $\frac{1}{2n} < \delta$

$$\frac{1}{4n} < \frac{1}{2n} < \delta$$



$$x_1 = \frac{-5}{2} + \frac{1}{2n}$$

$$x_2 = \frac{-5}{2} + \frac{1}{4n}$$

$$f(x_1) = \frac{1}{2x_1+5} = \frac{1}{2\left(\frac{-5}{2}+\frac{1}{2n}\right)+5}$$

$$= \frac{1}{-5+\frac{1}{n}+5} = n.$$

$$f(x_2) = \frac{1}{2x_2+5} = \frac{1}{2\left(\frac{-5}{2}+\frac{1}{4n}\right)+5}$$

$$= \frac{1}{-5+\frac{1}{2n}+5} = 2n.$$

$$|f(x_1) - f(x_2)| = |n-2n| = |-n| = n. > 1. > \epsilon$$

$[0<\epsilon<1]$.

$$|f(x_1) - f(x_2)| > \epsilon \quad \text{for } |x - 5/2| < \delta$$

By Cauchy Criterion.

$\lim_{x\to -5/2} \frac{1}{2x+5}$ does not exist.

H.W

$$\lim_{x \to -3/2} \frac{1}{2x+3} \text{ does not exist.}$$