Limit and Continuity

Example

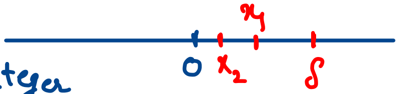
Show that $\lim\limits_{x \to 0} \sin \frac{1}{x}$ does not exist.

Proof: $\lim\limits_{x \to 0} \sin \frac{1}{x}$

Let $\epsilon > 0$ be however small $0 < \epsilon < 1$

$\delta > 0$ be any +ve real No.

Let $\frac{1}{n\pi} < \delta$ for n is +ve integer



$$x_1 = \frac{1}{2n\pi}$$

$$x_2 = \frac{1}{2n\pi + \pi/2}$$

$$f(x_1) = \sin\left(\frac{1}{x_1}\right) = \sin\left(\frac{1}{1/2n\pi}\right) = \sin 2n\pi$$

$$= 0$$ [zero occur at multiple of π]

$$f(x_2) = \sin\left(\frac{1}{x_2}\right) = \sin\left(\frac{1}{\frac{1}{2n\pi+\pi/2}}\right) = \sin\left(2n\pi + \pi/2\right)$$

$$= \sin \pi/2 = 1.$$

$$|f(x_1) - f(x_2)| = |0-1| = |-1| = 1 > \epsilon$$

$$[0 < \epsilon < 1]$$

$$\Rightarrow |f(x_1) - f(x_2)| > \epsilon \text{ for } |x| < \delta$$

$\therefore$ By Cauchy Criterion.

$\lim\limits_{x \to 0} \sin \frac{1}{x}$ does not exist.