Limit and Continuity

Example

Prove that $\lim_{x \to c^-} \frac{1}{x-c}$ does not exist.

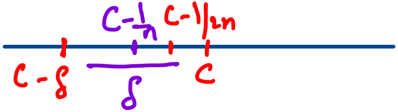
Proof

Let $\epsilon > 0$ be however small.

$0 < \epsilon < 1.$

Let $\delta > 0$ be any real No.

Let $\frac{1}{n} < \delta$ be any +ve integer



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$$\frac{1}{2n} < \frac{1}{n} < \delta$$

$$x_1 = c - \frac{1}{n}$$

$$x_2 = c - \frac{1}{2n}$$

$$f(x_1) = \frac{1}{x_1 - c} = \frac{1}{c - \frac{1}{n} - c} = -n.$$

$$f(x_2) = \frac{1}{x_2 - c} = \frac{1}{c - \frac{1}{2n} - c} = -2n.$$

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$$|f(x_1) - f(x_2)| = |-n - (-2n)|$$

$$= |-n + 2n| = n \geq 1 > \epsilon$$

$$\Rightarrow |f(x_1) - f(x_2)| > \epsilon \quad \text{for } x \in (c-\delta, c) \quad \{0 < \epsilon < 1\}$$

$\therefore$ By Cauchy Criterion

$\lim\limits_{x \to c^-} \dfrac{1}{x-c}$ does not exist.

Hence Proved.

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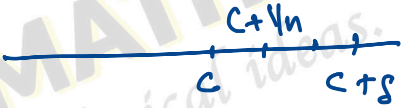
$$\lim_{x \to c^+} \frac{1}{x-c} \text{ does not exist.}$$



$$\frac{1}{n} < \delta$$

$$x_1 = c + \frac{1}{n}$$

$$x_2 = c + \frac{1}{2n}$$

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