Limit and Continuity

Example

Let $f(x) = \begin{cases} 2+x & : x \text{ is Rational} \\ 2.5-x & : x \text{ is irrational} \end{cases}$

then Prove that $\lim_{x \to 0.25} f(x) = 2.25$.

Proof $|f(x) - l| = |2 + x - 2.25|$

$= |x - 0.25|$ [x is Rational]

$|f(x) - l| = |2.5 - x - 2.25|$

$$= |0.25 - x| \qquad [x \text{ is irrational}]$$

$$= |-(x - 0.25)|$$

$$= |x - 0.25|$$

$$|f(x) - l| < \epsilon$$

$$|x - 0.25| < \epsilon \quad = \delta$$

$$|x - 0.25| < \delta$$

for $|f(x) - l| < \epsilon$ we have found a δ to s.t.

$$|x - 0.25| < \delta$$

Hence $\lim_{x \to 0.25} f(x) = 2.25$

Hence Proved.