

Limit and Continuity

Example

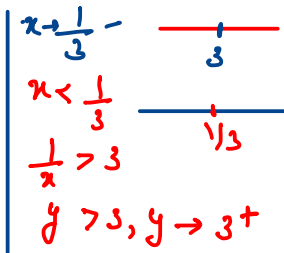
If $f(x) = x \left[\frac{1}{x} \right]$, then show that

$\lim_{x \rightarrow 1/3} f(x)$ does not exist.

Sol.

$$\text{Let } \frac{1}{x} = y.$$

$$\lim_{x \rightarrow 1/3^-} f(x) = \lim_{y \rightarrow 3^+} \frac{1}{y} [y]$$



$$= \lim_{y \rightarrow 3^+} \frac{1}{y} \cdot \lim_{y \rightarrow 3^+} [y]$$

$$= \lim_{\delta \rightarrow 0} \frac{1}{3+\delta} \cdot 3 = \frac{1}{3} \cdot 3 = 1.$$

$$\left[\begin{array}{l} \because \lim_{x \rightarrow a^+} f(x) = \lim_{\delta \rightarrow 0} f(a+\delta) \\ \lim_{x \rightarrow a^+} [x] = a. \end{array} \right]$$

$$\lim_{x \rightarrow 1/3^-} f(x) = 1. \quad \checkmark \quad \text{--- ①}$$

$$\lim_{x \rightarrow \frac{1}{3}^+} f(x)$$

$$\begin{array}{l} x \rightarrow 1/3^+ \\ x > \frac{1}{3} \\ \frac{1}{x} < 3 \\ y < 3. \\ y \rightarrow 3^- \end{array}$$

$$= \lim_{y \rightarrow 3^-} \left(\frac{1}{y} \right) [y]$$

$$= \lim_{y \rightarrow 3^-} \frac{1}{y} \cdot \lim_{y \rightarrow 3^-} [y]$$

$$= \lim_{\delta \rightarrow 0} \frac{1}{3-\delta} (3-1)$$

$$= \frac{1}{3} (2) = \frac{2}{3}$$

$$\lim_{x \rightarrow a^-} f(x) = \lim_{\delta \rightarrow 0} f(a-\delta)$$

$$\lim_{x \rightarrow a^-} [x] = a-1$$

$$\lim_{x \rightarrow \frac{1}{3}^+} f(x) = \frac{2}{3} \quad \text{--- (1)}$$

from (1) and (2)

$$\lim_{x \rightarrow \frac{1}{3}^-} f(x) \neq \lim_{x \rightarrow \frac{1}{3}^+} f(x)$$

$\Rightarrow \lim_{x \rightarrow \frac{1}{3}} f(x)$ does not exist.

Hence Proved.