

Limit and Continuity

Example

Let $f(x) = \frac{1-x}{1+x}$, $0 < x < 1$ find a real number $\delta > 0$ s.t.

$$|f(x) - \frac{1}{3}| < \frac{1}{10} \quad \text{for} \quad |x - \frac{1}{2}| < \delta$$

Sol.

$$f(x) = \frac{1-x}{1+x} \quad 0 < x < 1$$

$$l = \frac{1}{3}.$$

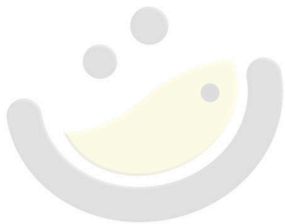
$$|f(x) - L| = \left| \frac{1-x}{1+x} - \frac{1}{3} \right|$$

$$= \left| \frac{3-3x-1-x}{3(1+x)} \right|$$

$$= \left| \frac{-4x+2}{3(1+x)} \right|$$

$$= \frac{4}{3(1+x)} \left| -x + \frac{1}{2} \right|$$

$$= \frac{4}{3(1+x)} \left| x - \frac{1}{2} \right| \quad - \textcircled{1}$$



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$$\underline{\text{Now}} \quad |f(x) - 1/3| < \frac{1}{10}$$

$$= \frac{4}{3(1+x)} |x - 1/2| < \frac{1}{10} \quad \text{for } |x - 1/2| < \delta \quad [\text{from } \textcircled{1}]$$

$$\Rightarrow |x - 1/2| < \frac{3(1+x)}{40} \quad \text{for } |x - 1/2| < \delta$$

$$\text{let } \delta = 3/40$$

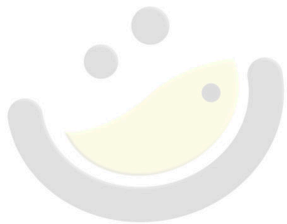
$$\text{So } |x - 1/2| < \frac{3}{40}$$

$$|x - 1/2| < \frac{3}{40} (1+x)$$

$$\left[\begin{array}{l} 0 < x < 1 \\ 1 < x+1 < 2 \end{array} \right]$$

$$\frac{4}{3(1+x)} |x - 1/2| < \frac{1}{10}$$

$$\Rightarrow \delta = 3/40$$



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