Limit and Continuity

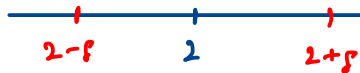**Example**

Evaluate $\lim\limits_{x \to 2} f(x)$ if it exist

where $f(x) = \begin{cases} x - \{x\} & x < 2 \\ 4 & x = 2 \\ 3x - 5 & x > 2 \end{cases}$

$\lim\limits_{x \to 2^-} x - \{x\}$

$= \lim\limits_{x \to 2^-} x - \lim\limits_{x \to 2^-} \{x\}$

2-δ   2   2+δ

$$= \lim_{\delta \to 0} (2-\delta) - (2-1)$$

$$\left[\because \lim_{x \to a^-} f(x) = \lim_{\delta \to 0} (a-\delta) \right]$$

$$= 2 - 1 = 1.$$

$$\left[\because \lim_{x \to a^-} [x] = (a-1) \right]$$

$$\lim_{x \to 2^-} f(x) = 1. \quad \text{—①}$$

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (3x-5)$$

$$= \lim_{\delta \to 0} [3(2+\delta)-5]$$

$$\left[\because \lim_{x \to a^+} f(x) = \lim_{\delta \to 0} f(a+\delta) \right]$$

$$= 3(2) - 5 = 1.$$

$$\lim_{x \to 2^+} f(x) = 1 \qquad ——\textcircled{11}$$

from ① and ⑪

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = 1.$$

$$\lim_{x \to 2} f(x) = 1.$$