Limit and Continuity

Example

By use of definition of limit, show that

$$\lim_{x \to 3} \frac{x^2 - 9}{x - 3} = 6$$

Proof $f(x) = \frac{x^2 - 9}{x - 3}$ $\quad l = 6$

$$|f(x) - l| = \left| \frac{x^2 - 9}{x - 3} - 6 \right| = \left| \frac{(x+3)(x-3)}{x - 3} - 6 \right|$$

$[a^2 - b^2 = (a+b)(a-b)]$

$|f(x)-l| = |x-3|$

Now $|f(x)-l| < \epsilon$ when

$|x-3| < \epsilon = \delta$

$\Rightarrow$ By def of limit

$$\lim_{x \to 3} f(x) = 6$$

$$\lim_{x \to 3} \frac{x^2-9}{x-3} = 6$$

Hence proved.