Limit and Continuity

Example

By using definition of limit show that $\lim_{x \to c} (x-c) \sin \frac{1}{x-c} = 0$

Proof $f(x) = (x-c) \sin \frac{1}{x-c}$ $\quad l = 0.$

$$|f(x) - l| = \left|(x-c) \sin \frac{1}{x-c} - 0\right| \leq |x-c|$$

$$\left[\because \left|\sin \frac{1}{x-c}\right| \leq 1\right]$$

$|f(x)-l| < \epsilon$ when $|x-c| < \epsilon$

$-1 \leq \sin\theta \leq 1$.

$|\sin\theta| \leq 1$.

$\Rightarrow |f(x)-l| < \epsilon$ whenever $0 < |x-c| < \epsilon = \delta$

$\therefore$ By definition of limit

$$\lim_{x \to c} f(x) = 0$$

$$\lim_{x \to c} (x-c) \sin \frac{1}{x-c} = 0$$

H.W. $\lim_{x \to c} (x-c) \cos \frac{1}{x-c} = 0$