Calculus- Lecture 28

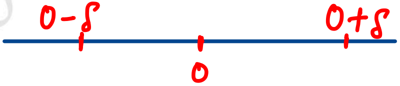
Limit and Continuity

Example

Prove that $\lim\limits_{x \to 0} \dfrac{x}{|x| + x^2}$ does not exist.

Proof

$$\lim_{x \to 0} \frac{x}{|x| + x^2}$$



$$\lim_{x \to 0^-} \frac{x}{|x| + x^2} = \lim_{\delta \to 0} \frac{0 - \delta}{|0-\delta| + (0-\delta)^2}$$

$\left[\because \text{ when } x \to 0^- \quad \delta \to 0\right]$

$$\lim_{\delta \to 0} \frac{-\delta}{\delta + \delta^2} = \lim_{\delta \to 0} \frac{-\delta}{\delta + \delta^2} = \lim_{\delta \to 0} \frac{-\delta}{\delta(1+\delta)}$$

$$\lim_{\delta \to 0} \frac{-1}{1+\delta} = -1$$

$$\lim_{x \to 0^-} f(x) = -1. \qquad \text{---} (1)$$

$$\lim_{x \to 0^+} \frac{x}{|x| + x^2}$$

$$\lim_{\delta \to 0} \frac{0+\delta}{|0+\delta| + (0-\delta)^2} = \lim_{\delta \to 0} \frac{\delta}{\delta + \delta^2}$$

$$= \lim_{\delta \to 0} \frac{\delta}{\delta(1+\delta)} = \lim_{\delta \to 0} \frac{1}{1+\delta}$$

$$= 1.$$

$$\lim_{x \to 0^+} f(x) = 1. \quad \text{— (II)}$$

from (I) and (II)

$$\lim_{x \to 0^-} f(x) \neq \lim_{x \to 0^+} f(x)$$

$\lim_{x \to 0} f(x)$ does not exist

Hence $\lim_{x \to 0} \frac{x}{|x| + x^2}$ does not exist.