

Limit and Continuity

Example

Prove that $\lim_{x \rightarrow 0} \frac{x}{|x| + x^2}$ does not exist.

Proof

$$\lim_{x \rightarrow 0} \frac{x}{|x| + x^2}$$



$$\lim_{x \rightarrow 0^-} \frac{x}{|x| + x^2} = \lim_{\delta \rightarrow 0} \frac{0 - \delta}{|0 - \delta| + (0 - \delta)^2} \left[\begin{array}{l} \because \text{when } x \rightarrow 0^- \\ \delta \rightarrow 0 \end{array} \right]$$

$$\lim_{\delta \rightarrow 0} \frac{-\delta}{\delta + \delta^2} = \lim_{\delta \rightarrow 0} \frac{-\delta}{\delta(1+\delta)} = \lim_{\delta \rightarrow 0} \frac{-1}{1+\delta}$$

$$\lim_{\delta \rightarrow 0} \frac{-1}{1+\delta} = -1$$

$$\lim_{x \rightarrow 0^-} f(x) = -1.$$

—①

$$\lim_{x \rightarrow 0^+} \frac{x}{|x| + x^2}$$

$$\begin{aligned} \lim_{\delta \rightarrow 0} \frac{0 + \delta}{(0 + \delta) + (0 - \delta)^2} &= \lim_{\delta \rightarrow 0} \frac{\delta}{\delta + \delta^2} \\ &= \lim_{\delta \rightarrow 0} \frac{\delta}{\delta(1 + \delta)} = \lim_{\delta \rightarrow 0} \frac{1}{1 + \delta} \\ &= 1. \end{aligned}$$

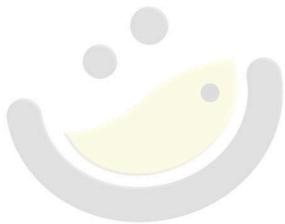
$$\lim_{x \rightarrow 0^+} f(x) = 1. \quad \text{--- (i)}$$

from (i) and (ii)

$$\lim_{x \rightarrow 0^-} f(x) \neq \lim_{x \rightarrow 0^+} f(x)$$

$\lim_{x \rightarrow 0} f(x)$ does not exist

hence $\lim_{x \rightarrow 0} \frac{x}{|x| + x^2}$ does not exist.



OMG { MATHS }
The poetry of logical ideas.