

Chapter 1 - Real Numbers

Theorem 1.2 (Fundamental Theorem of Arithmetic) : *Every composite number can be expressed (factorised) as a product of primes, and this factorisation is unique, apart from the order in which the prime factors occur.*

$$\begin{array}{r} 14 \\ 7 \overline{) 14} \\ \underline{2} \\ 2 \\ \underline{1} \\ 1 \end{array}$$

7×2

$$\begin{array}{r} 14 \\ 2 \overline{) 14} \\ \underline{7} \\ 7 \\ \underline{1} \\ 1 \end{array}$$

2×7

$$\begin{array}{r} 36 \\ 2 \overline{) 36} \\ \underline{18} \\ 3 \\ \underline{9} \\ 3 \\ \underline{3} \\ 1 \end{array}$$

$2 \times 2 \times 3 \times 3$
 $(2)^2(3)^2$

$$\begin{array}{r} 36 \\ 3 \overline{) 36} \\ \underline{12} \\ 3 \\ \underline{6} \\ 2 \\ \underline{2} \\ 1 \end{array}$$

$3 \times 2 \times 3 \times 2$
 $3^2 2^2$

$$\begin{array}{r} 36 \\ 2 \overline{) 36} \\ \underline{18} \\ 3 \\ \underline{6} \\ 3 \\ \underline{2} \\ 1 \end{array}$$

$2 \times 3 \times 3 \times 2$
 $3^2 2^2$

The prime factorisation of a natural number is unique, except for the order of its factors.

$$\begin{array}{r|l} 2 & 24 \\ \hline 2 & 12 \\ \hline 2 & 6 \\ \hline 2 & 3 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$$\underline{2 \times 2 \times 2 \times 3}$$

$$\begin{array}{r|l} 3 & 24 \\ \hline 2 & 8 \\ \hline 2 & 4 \\ \hline 2 & 2 \\ \hline & 1 \end{array}$$

$$\underline{3 \times 2 \times 2 \times 2}$$

1. Express each number as a product of its prime factors:

✓ (i) 140

$$\begin{array}{r|l} 2 & 140 \\ \hline 2 & 70 \\ \hline 7 & 35 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

$$2 \times 2 \times 7 \times 5$$

✓ (ii) 156

$$\begin{array}{r|l} 2 & 156 \\ \hline 2 & 78 \\ \hline 3 & 39 \\ \hline 13 & 13 \\ \hline & 1 \end{array}$$

$$2 \times 2 \times 3 \times 13$$

(iii) 3825

(iv) 5005

(v) 7429

Example 5 : Consider the numbers 4^n , where n is a natural number. Check whether there is any value of n for which 4^n ends with the digit zero.

Sol. Let 4^n ends with 0 for any no. n .

$$4^n = \underline{\quad 0 \quad}$$

\Rightarrow Prime factor of 4^n is 5 \rightarrow ①

Now $4^n = (2 \times 2)^n$

But By fundamental theorem prime factors are unique.
Which is contradiction of ① \Rightarrow there is no natural no for which 4^n ends with 0.

5. Check whether 6^n can end with the digit 0 for any natural number n .

Let 6^n ends with 0 for any natural no.

$$6^n = \underline{\quad\quad\quad} 0$$

$\Rightarrow 6^n$ has one prime factor 5 - ①

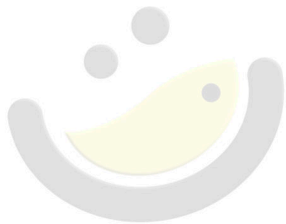
$$6^n = (2 \times 3)^n$$

By fundamental theorem prime factors
of a No. is unique.

Which is contradiction of ①

$$\begin{array}{r} 2 \overline{) 6} \\ \underline{3} \\ 3 \\ \underline{3} \\ 0 \end{array}$$

⇒ there is no natural number n for which 6^n ends with 0.



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The poetry of logical ideas.