Calculus- Lecture 17

Limit and Continuity

Cauchy Criterion

$\lim_{x \to a} f(x)$ exists finitely if given $\epsilon > 0$, however small, there exist a +ve real no. δ s.t.

$$|f(x_1) - f(x_2)| < \epsilon \text{ for } 0 < |x_i - a| < \delta$$

where $i = 1, 2$

$f(x) = \dfrac{x^2-1}{x-1} \quad x \neq 1$

x	.9	.99	.999	.9999
$f(x)$	1.9	1.99	1.999	1.9999

x	1.1	1.01	1.001	1.0001
$f(x)$	2.1	2.01	2.001	2.0001

Proof Let $\lim_{x \to a} f(x) = l$ [exists finitely]

$\therefore$ for $\epsilon > 0$ $\exists$ +ve real no. δ s.t.

$|f(x_1) - l| < \epsilon$ for $|x_1 - a| < \delta$ —(1)

also $|f(x_2)-l|<\epsilon$ for $|x_2-a|<\delta$ —(ii)

Now $|f(x_1)-f(x_2)| = |f(x_1)-f(x_2)+l-l|$

$= |\{f(x_1)-l\}+\{l-f(x_2)\}|$

$\leq |f(x_1)-l|+|l-f(x_2)|$ [$|a+b|\leq|a|+|b|$]

$= |f(x_1)-l|+|f(x_2)-l|$

$< \epsilon+\epsilon$ [from (i) & (ii)]

$= 2\epsilon$ for $|x_1-a|<\delta$

$|x_2-a|<\delta$

$= \epsilon'$

$|f(x_1)-f(x_2)|<\epsilon'$ for $|x_1-a|<\delta$ and $|x_2-a|<\delta$