Calculus 1

If A_1 and A_2 are two bounded subsets of R, then the set $A_1+A_2=\{x+y: x\in A_1 \text{ and } y\in A_2\}$ is also bounded. Further if $u_1=\text{Sup } A_1, u_2=\text{Sup } A_2$ then $\text{Sup.}(A_1+A_2)=u_1+u_2$

Sol.

A_1 is Bounded.

$\Rightarrow l_1 \leqslant x \leqslant u_1 \quad \forall x \in A_1 \quad -(1)$

A_2 is Bounded.

$$\Rightarrow l_2 \leq y \leq u_2 \quad \forall \; y \in A_2 \quad \text{—(ii)}$$

from (i) and (ii)

$$l_1 + l_2 \leq x + y \leq u_1 + u_2 \quad \forall \; x \in A_1 \text{ and } y \in A_2$$

$$l_1 + l_2 \leq x + y \leq u_1 + u_2 \quad \forall \; x + y \in A_1 + A_2$$

$\Rightarrow$ $A_1 + A_2$ is Bounded.

Let z is any element of $A_1 + A_2$

then z will be in the form of $x + y \; \forall \, x \in A_1,$
$y \in A_2$

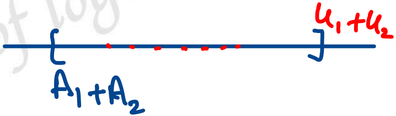
Now

$x \leq u_1 \quad [\because u_1 = \sup A_1]$

$y \leq u_2 \quad [\because u_2 = \sup A_2]$

$\therefore \; z = x + y \leq u_1 + u_2$

$\Rightarrow$ $u_1 + u_2$ is upper Bound of $A_1 + A_2$



u_1 is $\sup A_1$

$u_1 - \epsilon/2 < x \leq u_1$ — (II)

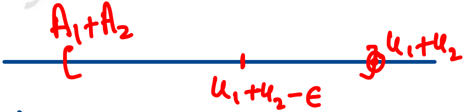
u_2 is $\sup A_2$

$$\Rightarrow u_2 - \epsilon/2 < y \leq u_2 \quad \text{— (IV)}$$

Add (III) & (IV)

$$u_1 + u_2 - \epsilon < x + y \leq u_1 + u_2$$

So By def.



$u_1 + u_2$ is $\sup(A_1 + A_2)$

$$\Rightarrow \sup A_1 + \sup A_2 = \sup(A_1 + A_2)$$

A_1 and A_2 be two subsets of R which are Bounded Below

Prove that $A_1 + A_2 = \left\{ x+y \;\middle|\; \begin{array}{l} x \in A_1 \\ y \in A_2 \end{array} \right\}$

is Bounded Below.

Also Prove that

$$\text{g.l.b. of } (A_1 + A_2) = \text{g.l.b of } A_1 + \text{g.l.b of } A_2$$