

Calculus 1

Show that the set is Unbounded

$$S = \left\{ \frac{3-x}{1-x} ; x > 0, x \neq 1 \right\}$$

$$y = \frac{3-x}{1-x} \quad x > 0, x \neq 1.$$

$$y(1-x) = 3-x$$

$$y - yx = 3 - x$$

$$-yx + x = 3 - y$$

$$x(1-y) = 3 - y.$$

$$x = \frac{3-y}{1-y} > 0 \quad -\textcircled{1}$$

Case I - $1-y > 0$

$$-y > -1$$

$$y < 1$$

from $\textcircled{1}$ $1-y > 0.$

$$\Rightarrow 3-y > 0$$

$$-y > -3$$

$$y < 3.$$

$$\Rightarrow y = (-\infty, 1) \quad -\textcircled{11}$$

Case II

$$1-y < 0$$

$$-y < -1$$

$$y > 1.$$

from ①

$$\frac{3-y}{1-y} > 0$$

$$\Rightarrow \begin{aligned} 3-y &> 0 \\ -y &< -3 \end{aligned}$$

$$y > 3.$$

$$y = (3, \infty) \quad - \text{III}$$

from ⑦ and ⑪

$$y = (-\infty, 1) \cup (3, \infty)$$

$\Rightarrow S$ is unbounded.