

Calculus 1

Find glb and lub of the set.

$$y = \sin^2 x + \cos^4 x \quad : x \in \mathbb{R}$$

Sol.

$$y = \sin^2 x + \cos^4 x$$

$$= \sin^2 x + \cos^2 x \cdot \cos^2 x$$

$$= \sin^2 x + \cos^2 x (1 - \sin^2 x) \quad \left[\cos^2 \theta = 1 - \sin^2 \theta \right]$$

$$= \sin^2 x + \cos^2 x - \cos^2 x \sin^2 x$$

$$= 1 - (\cos x \sin x)^2 \quad \left[\sin^2 \theta + \cos^2 \theta = 1 \right]$$

$$= 1 - (2 \cos x \sin x)^2 \times \frac{1}{4}$$

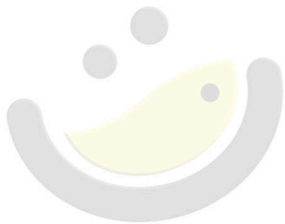
$$= 1 - (\sin 2x)^2 \frac{1}{4} \quad [2 \sin \theta \cos \theta = \sin 2\theta]$$

$$y = 1 - \frac{1}{4} \sin^2 2x \quad \text{--- ①}$$

$$0 \leq \sin^2 2x \leq 1$$

$$[\because 0 \leq \sin^2 \theta \leq 1]$$

$$0 \leq \frac{1}{4} \sin^2 2x \leq \frac{1}{4}$$



$$0 \leq \frac{-1}{4} \sin^2 2x \leq \frac{-1}{4}$$

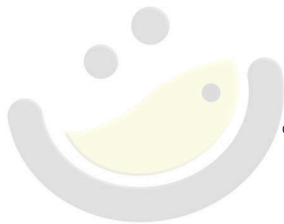
$$1 \leq 1 - \frac{1}{4} \sin^2 2x \leq 1 - \frac{1}{4}$$

$$1 \leq y \leq \frac{3}{4}$$

$$\frac{3}{4} \leq y \leq 1.$$

$$\text{l.u.b} = 1$$

$$\text{g.l.b} = \frac{3}{4}$$



OMG! MATHS }
The poetry of logical ideas.