Calculus 1

Show that the following sets are bounded. Also Find glb and lub of the set.

(i) $\{(\sin x + \cos x)^2 : 0 \leq x \leq \pi\}$

(ii) $\{2\sin x - 3\cos x\}$

(i) Sol.

$y = (\sin x + \cos x)^2 \qquad 0 \leq x \leq \pi.$

$y = \sin^2 x + \cos^2 x + 2\sin x \cos x \qquad [(a+b)^2 = a^2 + b^2 + 2ab]$

$= 1 + \sin 2x \qquad [\because \sin^2\theta + \cos^2\theta = 1]$

$[2\sin\theta\cos\theta = \sin 2\theta]$

$$y = 1 + \sin 2x.$$

$$0 \leq x \leq \pi$$

$$0 \leq 2x \leq 2\pi$$

$$-1 \leq \sin 2x \leq 1.$$

$$-1+1 \leq 1+\sin 2x \leq 1+1$$

$$0 \leq y \leq 2.$$

$\therefore \quad y = (\sin x + \cos x)^2 \quad \forall \quad 0 \leq x \leq \pi.$

is Bounded.

$l.u.b = 2.$

$g.l.b = 0$

(ii) $y = 2\sin x - 3\cos x.$

$2 = r\cos\alpha \quad - (i)$

$3 = r\sin\alpha. \quad - (ii)$

By squaring and adding (i) & (ii)

$$4 + 9 = r^2 \cos^2\alpha + r^2 \sin^2\alpha$$

$$13 = r^2 (\cos^2\alpha + \sin^2\alpha)$$

$$13 = r^2 \qquad [\because \sin^2\theta + \cos^2\theta = 1]$$

$$\Rightarrow \quad r = \sqrt{13}.$$

$$y = r\cos\alpha \sin x - r\sin\alpha \cos x$$

$$= r\,[\sin x \cos\alpha - \cos x \sin\alpha] \qquad [\sin(a-b) = \sin a \cos b - \cos a \sin b]$$

$$= r \sin(x-\alpha)$$

$$= \sqrt{13}\sin(x-\alpha)$$

$$-1 \leqslant \sin(x-\alpha) \leqslant 1.$$

$$-\sqrt{13} \leqslant \sqrt{13}\sin(x-\alpha) \leqslant \sqrt{13}.$$

$$-\sqrt{13} \leqslant y \leqslant \sqrt{13}.$$

$\Rightarrow$ set $y = 2\sin x - 3\cos x$. is Bounded.

l.u.b $= \sqrt{13}$.

g.l.b $= -\sqrt{13}$.

$y = a\sin x + b\cos x + c \quad : x \in \mathbb{R}$