

Calculus 1

Show that the following sets are bounded.
Also Find glb and lub of the set.

(i) $\{(\sin x + \cos x)^2 : 0 \leq x \leq \pi\}$

(ii) $\{2 \sin x - 3 \cos x\}$

(i) Sol. $y = (\sin x + \cos x)^2 \quad 0 \leq x \leq \pi.$

$$y = \sin^2 x + \cos^2 x + 2 \sin x \cos x$$

$$= 1 + \sin 2x$$

$$\left[\begin{array}{l} (a+b)^2 = a^2 + b^2 \\ + 2ab \\ \therefore \sin^2 0 + \cos^2 0 = 1 \end{array} \right]$$

$$[2 \sin \theta \cos \theta = \sin 2\theta]$$

$$y = 1 + \sin 2x.$$

$$0 \leq x \leq \pi$$

$$0 \leq 2x \leq 2\pi$$

$$-1 \leq \sin 2x \leq 1.$$

$$-1 + 1 \leq 1 + \sin 2x \leq 1 + 1$$

$$0 \leq y \leq 2.$$

$$\therefore y = (\sin x + \cos x)^2 \quad \forall \quad 0 \leq x \leq \pi.$$

is Bounded.

$$l.u.b = 2.$$

$$g.l.b = 0$$

$$(ii) \quad y = 2 \sin x - 3 \cos x.$$

$$2 = r \cos \alpha \quad - \textcircled{1}$$

$$3 = r \sin \alpha. \quad - \textcircled{II}$$

By squaring and adding $\textcircled{1} + \textcircled{II}$

$$4 + 9 = r^2 \cos^2 \alpha + r^2 \sin^2 \alpha$$

$$13 = r^2 (\cos^2 \alpha + \sin^2 \alpha)$$

$$13 = r^2 \quad [\because \sin^2 \theta + \cos^2 \theta = 1]$$

$$\Rightarrow r = \sqrt{13}$$

$$y = r \cos \alpha \sin x - r \sin \alpha \cos x$$

$$= r [\sin x \cos \alpha - \cos x \sin \alpha] \quad \left[\begin{array}{l} \sin(a-b) = \sin a \cos b \\ - \cos a \sin b \end{array} \right]$$

$$= r \sin(x - \alpha)$$

$$= \sqrt{13} \sin(x - \alpha)$$

$$-1 \leq \sin(x-2) \leq 1.$$

$$-\sqrt{13} \leq \sqrt{13} \sin(x-2) \leq \sqrt{13}.$$

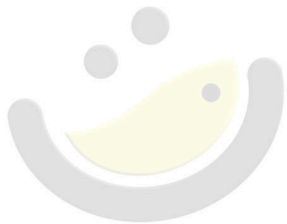
$$-\sqrt{13} \leq y \leq \sqrt{13}.$$

\Rightarrow set $y = 2 \sin x - 3 \cos x$ is Bounded.

$$\text{L. u. b} = \sqrt{13}.$$

$$\text{g. l. b} = -\sqrt{13}.$$

$$y = a \sin x + b \cos x + c \quad : x \in \underline{\underline{\mathbb{R}}}$$



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