Find glb and lub of the set

$S = 3\sin x + 4\cos x$ where $x \in R$.

$y = 3\sin x + 4\cos x$

$3 = r\cos\alpha$ — ①

$4 = r\sin\alpha$ — ②

By squaring and adding ① & ②

$9 + 16 = r^2(\cos^2\alpha + \sin^2\alpha)$ $\{\because (\cos^2\theta + \sin^2\theta) = 1\}$

$25 = r^2$

$r = 5$

$y = 3\sin x + 4\cos x$

$= r\cos\alpha \sin x + r\sin\alpha \cos x.$

$= r\left[\cos\alpha \sin x + \sin\alpha \cos x\right]$ $\left[\sin A\cos B + \cos A \sin B = \sin(A+B)\right]$

$= r\left[\sin(x+\alpha)\right]$

$= 5\sin(x+\alpha)$

$-1 \leq \sin(x+\alpha) \leq 1 \quad \forall x \in R$

$-5 \leq 5\sin(x+\alpha) \leq 5$

$-5 \leq y \leq 5$.

l.u.b = 5

g.l.b = -5.