

Calculus

Find glb and lub of the set

$$S = 3 \sin x + 4 \cos x \quad \text{where } x \in \mathbb{R}.$$

$$y = 3 \sin x + 4 \cos x$$

$$3 = r \cos \alpha \quad \text{--- (1)}$$

$$4 = r \sin \alpha \quad \text{--- (2)}$$

By squaring and adding (1) + (2)

$$9 + 16 = r^2 (\cos^2 \alpha + \sin^2 \alpha) \quad \left[\because (\cos^2 \theta + \sin^2 \theta) = 1 \right]$$
$$25 = r^2$$

$$r = 5$$

$$y = 3 \sin x + 4 \cos x$$

$$= r \cos \alpha \sin x + r \sin \alpha \cos x$$

$$= r [\cos \alpha \sin x + \sin \alpha \cos x]$$

$$\left[\begin{array}{l} \sin A \cos B + \\ \cos A \sin B = \\ \sin(A+B) \end{array} \right]$$

$$= r [\sin(x+\alpha)]$$

$$= 5 \sin(x+\alpha)$$

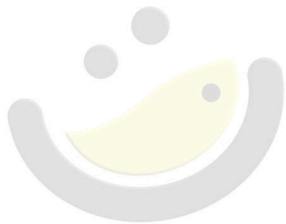
$$-1 \leq \sin(x+\alpha) \leq 1 \quad \forall x \in \mathbb{R}$$

$$-5 \leq 5 \sin(x+\alpha) \leq 5$$

$$-5 \leq y \leq 5.$$

$$\text{l.u.b} = 5$$

$$\text{g.l.b} = -5.$$



OMG { MATHS }

The poetry of logical ideas.