

THE RIEMANN-STIELTJES INTEGRAL

f is a constant function on $[a, b]$ defined by $f(x) = k$ and α is monotonically increasing on $[a, b]$ then show that $\int_a^b f d\alpha$ exists and

$$\int_a^b f d\alpha = k [\alpha(b) - \alpha(a)]$$

Proof

f is constant function

$$f(x) = k.$$

Now m_i and M_i are bounds of f in $[x_{i-1}, x_i]$ $1 \leq i \leq n$.

$$m_i = M_i = k$$

where $P = \{a = x_0, x_1, x_2, \dots, x_n = b\}$
is partition.

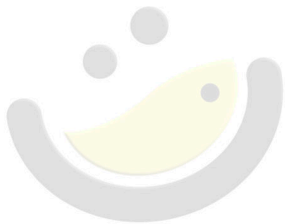
Now $L(p, f, \alpha) = \sum_{i=1}^n m_i \Delta \alpha_i$

$$= k \sum_{i=1}^n \Delta \alpha_i$$

$$= k [\alpha(x_1) - \alpha(x_0) + \alpha(x_2) - \alpha(x_1) + \dots + \alpha(x_{n-1}) - \alpha(x_{n-2}) + \alpha(x_n) - \alpha(x_{n-1})]$$

$$= k [\alpha(x_n) - \alpha(x_0)]$$

$$= k [\alpha(b) - \alpha(a)]$$



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$$\int_a^b f d\alpha = \sup [L(p, f, \alpha) : p \text{ is partition of } [a, b]]$$

$$\int_a^b f d\alpha = K [\alpha(b) - \alpha(a)] \quad \text{--- (1)}$$

$$U(p, f, \alpha)$$

$$= \sum_{i=1}^n M_i \Delta \alpha_i$$

$$= K \sum_{i=1}^n \Delta \alpha_i$$

$$= K [\alpha(x_n) - \alpha(x_0)]$$

$$= K [\alpha(b) - \alpha(a)]$$

$$\int_a^b f d\alpha = \inf [U(P, f, \alpha)] \quad P \text{ is partition of } [a, b]$$

$$\int_a^b f d\alpha = K [\alpha(b) - \alpha(a)] \quad \text{--- (11)}$$

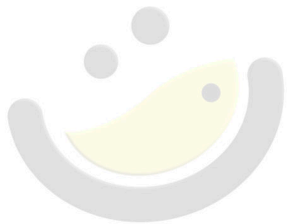
$$\int_a^b f d\alpha = \int_a^b f d\alpha \quad [\text{from (1) and (11)}]$$

$$\Rightarrow \int_a^b f d\alpha \text{ exists.}$$

$$\int_a^b f d\alpha = \int_a^b f d\alpha = \int_a^b f d\alpha$$

$$\int_a^b f d\alpha = k [\alpha(b) - \alpha(a)]$$

Hence proved.



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