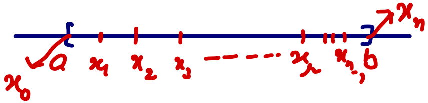
THE RIEMANN-STIELTJES INTEGRAL

Partition :- Let $[a,b]$ be given interval.

By a partition of $[a,b]$, we mean a finite set $P = \{a = x_0, x_1, \ldots, b = x_n\}$ of real numbers such that

$$x_0 < x_1 < x_2 \ldots \ldots < x_{n-1} < x_n$$

$[x_0, x_1]$; $[x_1, x_2]$; $[x_2, x_3]$; ---- $[x_{n-1}, x_n]$

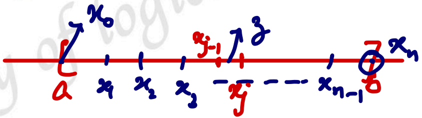


length of sub-intervals :- $\Delta x_i = x_i - x_{i-1}$

Norm of Partition:- length of sub-intervals of partition P is called norm of partition.

denoted By $\|P\|$ (or) $\mu(P)$

Refinement:-



$$P = \{a = x_0, x_1, x_2 \,--\, x_{j-1}, x_j, x_{n-1}, x_n = b\}$$

$$P_1 = \{a = x_0, x_1, x_2, \,----\, x_{j-1}, x_z, x_j \,\_\_\,..., x_{n-1}, x_n = b\}$$

$P \subset P_1 \quad \therefore P_1$ is Refinement of P.

$[1,4]$

$P = \{1, 2, 3, 4\}$

$P = \{1 = x_0, x_1, x_2, x_3 = 4\}$

$P_1 = \{1, 1.5, 2, 2.5, 3, 3.5, 4\}$

$P \subset P_1$

P_1 is Refinement of P.

Common Refinement: $P^* = P \cup P_1$ is Called Common Refinement of partitions P and P_1