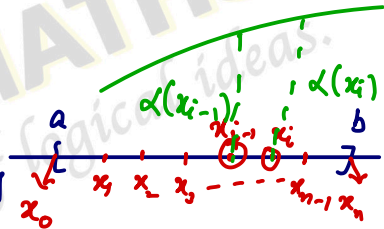


# THE RIEMANN-STIELTJES INTEGRAL

## Lower Riemann-Stieltjes Sum and Upper Riemann-Stieltjes Sum

Partition  $[a, b]$

$$P = \{a = x_0, x_1, x_2, \dots, x_n = b\}$$



Let  $\alpha$  is a real valued increasing function defined on  $[a, b]$

$$\Delta\alpha(x_i) = \alpha(x_i) - \alpha(x_{i-1})$$

$f$  is a real valued function on  $[a, b]$

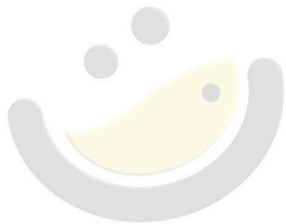
$$\begin{aligned} &> 0 \\ &[\alpha(x_i) > \alpha(x_{i-1})] \end{aligned}$$

$$m_i = \inf \{f(x) \mid x \in [x_{i-1}, x_i]\}$$

$$M_i = \sup \{f(x) \mid x \in [x_{i-1}, x_i]\}$$

$$L(P, f, \alpha) = \sum_{i=1}^n m_i \Delta x_i$$

$$U(P, f, \alpha) = \sum_{i=1}^n M_i \Delta x_i$$



OMG! MATHS }  
The poetry of logical ideas.