# THE RIEMANN-STIELTJES INTEGRAL

## Lower R-S Integral and Upper R-S Integral

$m\{\alpha(b)-\alpha(a)\} \leq L(P,f,\alpha) \leq U(P,f,\alpha) \leq M(\alpha(b)-\alpha(a))$

$M\{\alpha(b)-\alpha(a)\}$ is upper Bound of $L(P,f,\alpha)$

where $P$ is partition of $[a,b]$

By Order Completeness property.

sup of $L(P,f,\alpha)$ exist.

Sup value of $L(P,f,\alpha)$ over all partition of $[a,b]$ is called lower Riemann stieltjes integral.

denoted By $\int_a^b f\, d\alpha$ or $\int_a^b f(x)\, d\alpha(x)$

$$\int_a^b f\, d\alpha = \text{Sup}\ \{L(P, f, \alpha) : P \text{ is partition of } [a,b]\}$$

$$m\,[\alpha(b) - \alpha(a)] \leq U(P, f, \alpha)$$

$m[\alpha(b) - \alpha(a)]$ lower Bound of $U(P, f, \alpha)$

By order Completeness property.

infimum value exists.

$\inf(U(P, f, \alpha))$ is upper R-S integral.

$$\overline{\int_a^b} f\,d\alpha \quad \text{or} \quad \overline{\int_a^b} f(x)\,d\alpha(x)$$

$$\overline{\int_a^b} f\,d\alpha = \inf\{U(P,f,\alpha) : P \text{ is partition of } [a,b]\}$$

$$\underline{\int_a^b} f\,d\alpha = \overline{\int_a^b} f\,d\alpha.$$

then $f$ is said to by R-S integrable.

$$\int_a^b f\,d\alpha, \qquad f \in R(\alpha)$$