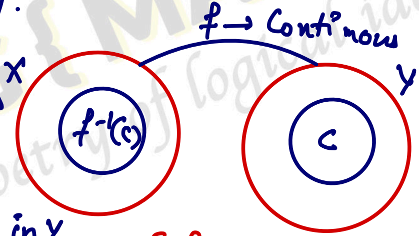
Limit and Continuity

Let X and Y are metric spaces. A function $f: X \to Y$ is Continous iff $f^{-1}(c)$ is closed set in X $\forall$ closed set c in Y.



Proof:- $f: X \to Y$ is Continous

c is closed set in Y.

$\Rightarrow (Y - c)$ is open set in Y.

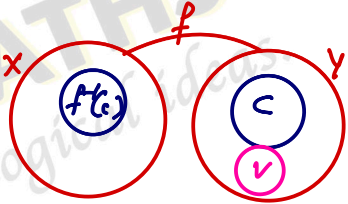
$\Rightarrow f^{-1}(Y - c)$ is open set in X.

$\Rightarrow f^{-1}(Y) - f^{-1}(c)$ is open in X.

[$f: X \to Y$ is continous iff $f^{-1}(U)$ is open set in X $\forall$ open set U in Y]

$\Rightarrow \quad X - f^{-1}(C)$ is open set in X.

$\Rightarrow \quad f^{-1}(C)$ is closed set in X.



Converse $f^{-1}(C)$ is closed set in X $\forall$ closed set C in Y.

Let V be an open set in Y

$\Rightarrow Y - V$ is closed set in Y.

$\Rightarrow f^{-1}(Y-V)$ is closed set in X.

$\Rightarrow f^{-1}(Y) - f^{-1}(V)$ is closed set in X.

$\Rightarrow \quad X - f^{-1}(V)$ is closed set in X

$\Rightarrow$ $f^{-1}(V)$ is open set in X.

$\Rightarrow$ $f: X \to Y$ is Continous.

[$f: X \to Y$ is Continous iff $f^{-1}(V)$ is open set in X $\forall$ open set V in Y]

Hence proved.