Calculus - 1

Properties of Real Numbers and Bounds : lecture 6

Archimedian Property Of Real Numbers

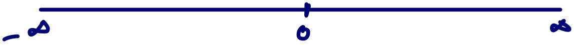
For given $a > 0$ and $b \in \mathbb{R}$ $\exists$ a natural number n s.t.

$na > b$

Proof

$b \in \mathbb{R}$.

(i) $b \leq 0$ (ii) $b > 0$



when $\underbrace{b \leq 0}$ $a > 0$

$na > b$ hold for all $n \in \mathbb{N}$.

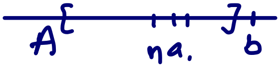
When $b > 0$

Let $na \leq b \quad \forall\, n \in \mathbb{N}$.

Now A set $A = \{na : n \in \mathbb{N},\ na \leq b\}$

A is Bounded above by b.

$\Rightarrow$ sup A exists.



$na \leq b$
$\Rightarrow$

$\Rightarrow$ Let $u = \sup A$.

$\Rightarrow \quad na \leq u. \quad \forall\, n \in \mathbb{N}$.

$\Rightarrow \quad (n+1)\,a \leq u \quad \forall\, n \in \mathbb{N}$.

$\Rightarrow \quad na + a \leq u \quad \forall\, n \in \mathbb{N}.$

$\Rightarrow \quad na \leq u - a \quad \forall\, n \in \mathbb{N}.$

$\Rightarrow \quad (u - a)$ is upper bound of A.

But $u - a < u = \sup A$.

which is a contradiction.

So our supposition is wrong.

Hence $na > b \quad \forall\, n \in \mathbb{N}$.

Hence Proved.