Uniform Continuity

Let X and Y are metric spaces. $f : X \to Y$ be uniformly continous if $\{x_n\}$ is Cauchy sequence in X. then $\{f(x_n)\}$ is Cauchy sequence in Y.

Proof

X, Y are metric spaces

$f : X \to Y$ is uniformly continous

By def.

for $\epsilon > 0$ $\exists$ $\delta > 0$ s.t.

$d_Y(f(x), f(y)) < \epsilon$ when $d_X(x, y) < \delta$ — (1)

$\{x_n\}$ is Cauchy sequence in X.

By def. of Cauchy sequence

for $\delta > 0$ $\exists$ $n_0 \in N$ s.t.

$d_X(x_n, x_m) < \delta$ for $m, n \geq n_0$

Now for $m, n \geq n_0$ $d_X(x_n, x_m) < \delta$

from 1

$d_Y(f(x_n), f(x_m)) < \epsilon$ for $m, n \geq n_0$

By def. of Cauchy sequence

$\{f(x_n)\}$ is a Cauchy sequence

Hence Proved