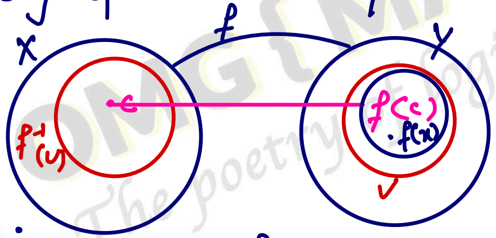
Continuity of a function

Let X and Y are metric spaces. A function $f: X \to Y$ is continous iff $f^{-1}(V)$ is open in X for every open set V in Y.



Proof: f is continous function

Let $c \in f^{-1}(V)$

$f(c) \in V$

V is an open set.

$\Rightarrow \exists\, B_Y(f(c), \epsilon) \subseteq V$ [By def. of open set]

f is continous at c.

By def. of Continuity.

for $\epsilon > 0$ $\exists\, \delta > 0$ s.t.

$d_Y(f(x), f(c)) < \epsilon$ when $d_X(x, c) < \delta$

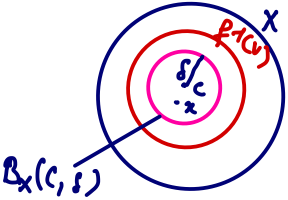
$\underbrace{x \in X}$.

Let $x \in B_X(c, \delta)$

$d_X(x, c) < \delta$

$\Rightarrow d_Y(f(x), f(c)) < \epsilon$

$\Rightarrow f(x) \in B_Y(f(c), \epsilon) \subseteq V$



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$\Rightarrow \quad f(x) \subseteq V$

$\Rightarrow \quad x \subseteq f^{-1}(V)$

$\Rightarrow \quad B_x(c, \delta) \subseteq f^{-1}(V)$

$\Rightarrow$ $f^{-1}(V)$ is open set.

Converse Part. $\quad f^{-1}(V)$ is open set in X when V is open in Y

T.P. $\quad f$ is Continous at c.

Now $\quad V$ is open set.

Let $B_Y(f(c), \epsilon) = V \quad — (1)$

$f^{-1}(V)$ is open set in X.

$$c \in f^{-1}(V)$$

$$\Rightarrow \quad B_X(c,\delta) \subseteq f^{-1}(V)$$

$$\Rightarrow \quad f(B_X(c,\delta)) \subseteq f(f^{-1}(V)) = V.$$

$$\Rightarrow \quad f(B_X(c,\delta)) \subseteq V$$

from (1)

$$\Rightarrow \quad f(B_X(c,\delta)) \subseteq B_Y(f(c),\epsilon)$$

$\Rightarrow$ f is continous at c. $\quad [\because c$ is a limit point$]$

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