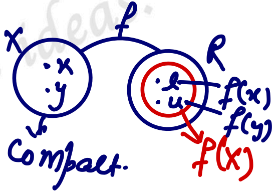
Continuity and Compactness

Maximum and Minimum Value thm.

$f: X \to R$ is Continous and X is Compact then f attains its Bounds in X.



Proof

$f: X \to R$ is Continous

X is Compact

Continous image of Compact set is Compact.

$\Rightarrow$ $f(X)$ is Compact.

$\Rightarrow$ $f(X)$ is Closed and Bounded in $\mathbb{R}$.

By Order Completeness property.

$$\left.\begin{array}{l}\inf(f(X)) \\ \sup(f(X))\end{array}\right] \text{ exists in } \mathbb{R}.$$

Let $l = \inf(f(X))$

$u = \sup(f(X))$

$l, u \in \overline{f(X)}$

$l, u \in f(X)$ $\left[\because \overline{f(X)} = f(X) \text{ as } f(X) \text{ is closed}\right]$

$l = f(x)$ $\quad u = f(y)$

for $x, y \in X$

$\Rightarrow f$ attains its Bounds in X.