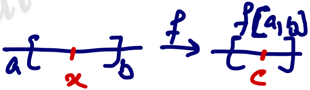
Continuity and Connectedness

Let $f: [a,b] \to R$ be continous function such that $f(a) < c < f(b)$ then $\exists$ $x \in (a,b)$ s.t. $f(x) = c$.

$[a,b] \longrightarrow R$



Proof $E \subseteq R$ is connected iff E is an interval

$\therefore$ $[a,b]$ is connected.

$\Rightarrow$ $f[a,b]$ is connected. [Continous image of connected set is connected]

$\Rightarrow$ $f[a,b]$ is interval.

$f(a)$ & $f(b)$ $\in f[a,b]$

$f(a) < c < f(b)$

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$\Rightarrow \quad c \in f[a,b].$

Now $f(x)=c$ for some $x \in [a,b]$

$f(x) \neq f(a) \quad \left[\because f(x) = c \neq f(a)\right]$

$\Rightarrow \quad x \neq a.$

$|||ly \quad x \neq b.$

$c = f(x)$ for some $x \in (a,b)$

Hence Proved.

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