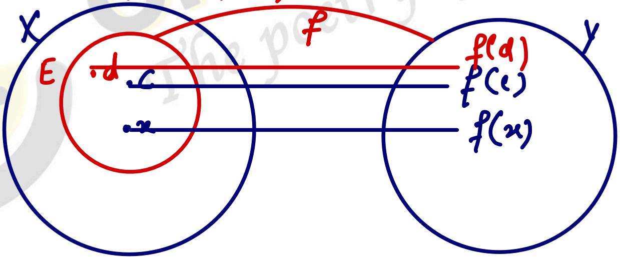
Continuity of a function

Let (X, d_1) and (Y, d_2) be metric spaces. $E \subseteq X$ and $f : E \rightarrow Y$ be a function. then f is said to be continous at $c \in E$ if

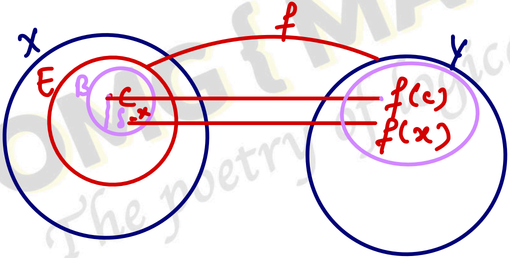
for $\epsilon > 0$ $\exists$ $\delta > 0$ s.t.

$d_2(f(x), f(c)) < \epsilon$ when $d_1(x, c) < \delta$



f is Continous at every point of E then f is said to be Continous function on E.

Case I when c is limit point of E.



$E \cap B(c, \delta) - \{c\} \neq \phi.$

f is Continous at point c iff for $\epsilon > 0$ $\exists$ $\delta > 0$ s.t.

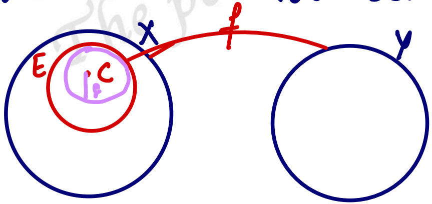
$d_2(f(x), f(c)) < \epsilon$ when $d_1(x, c) < \delta$ $\underline{x \in E}$.

$\underline{\text{iff}}$ $\lim\limits_{x \to c} f(x) = f(c)$

iff $x \in B_{d_1}(c, \delta) \Rightarrow f(x) \in B_{d_2}(f(c), \epsilon)$

iff $f(B_{d_1}(c, \delta)) \subseteq B_{d_2}(f(c), \epsilon)$

<u>Case II</u> when c is not limit point of E.



$E \cap B_{d_1}(c, \delta) - \{c\} = \phi$.

for $\epsilon > 0$ $\exists\ \delta > 0$

s.t. for $d_1(x, c) < \delta$

$$d_2(f(x), f(c)) < \epsilon$$ where $x = c$

[$\because$ c is not limit point

$B_{d_1}(x, c)$ Contains no point Other than c of E]

$$d_2(f(x), f(c))$$

$$= d_2(f(c), f(c))$$

$$= 0 < \epsilon$$

$$d_2(f(x), f(c)) < \epsilon$$

$\Rightarrow$ $f(x)$ is always Continous at c.