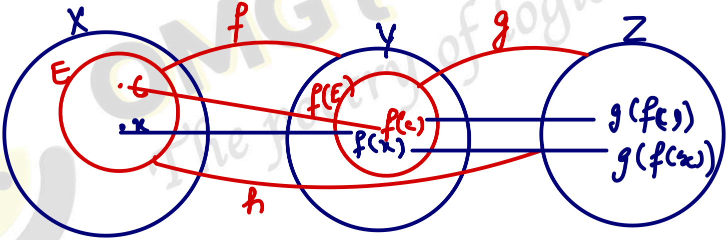
Continuity of a function

Let X, Y, Z be metric spaces, $E \subseteq X$, $f: E \to Y$, $g: f(E) \to Z$ are functions. If f is continous at $c \in E$ and g is continous at $f(c)$ then the composite map $h = g \circ f : E \to Z$ is continous at c.



Proof

g is continous at $f(c)$

By def. of continuity.

for $\epsilon > 0$ $\exists$ $\delta_1 > 0$ s.t.

$d_Z(g(f(x)), g(f(c))) < \epsilon$ when.

$d_Y(f(x), f(c)) < \delta_1$ —— (i)

f is continous at c.

By def of continuity.

$d_Y(f(x), f(c)) < \delta_1$ when —— (ii)

$d_X(x, c) < \delta$ where $x \in E$.

from (i) & (ii)

When $d_X(x, c) < \delta \Rightarrow d_Y(f(x), f(c)) < \delta_1$

$\Rightarrow d_Z[g(f(x)), g(f(c))] < \epsilon$

$\Rightarrow d_2[h(x), h(c)] < \epsilon$

when $d_X(x, c) < \delta$

$d_2[h(x), h(c)] < \epsilon$.

By def. of Continuity.

h is continous at point c.

Hence proved.